香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

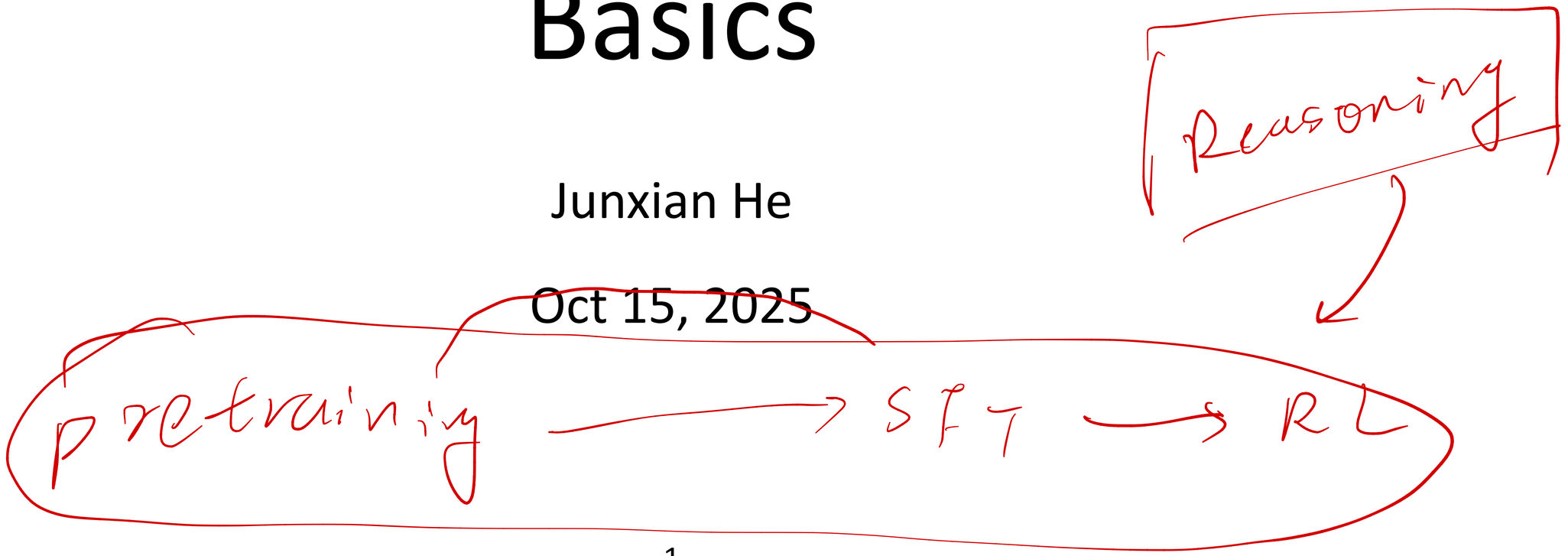
COMP 4901B

Large Language Models

Reinforcement Learning Basics

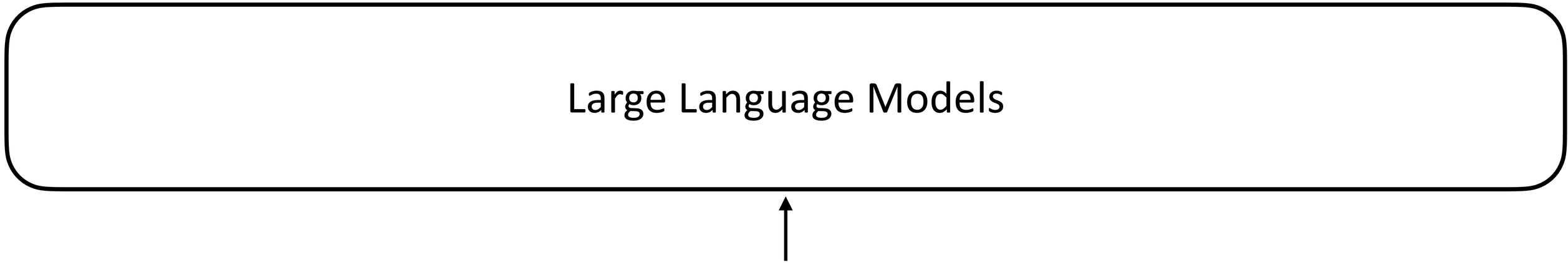
Junxian He

Oct 15, 2025



Review: Multi-Turn Instruction Tuning

<User> How are you today?\n<Assistant> I'm doing well, thank you! How can I help you? \n\n<User> Can you tell me a joke? \n<Assistant> Sure! Why did the math book look sad? <stop>



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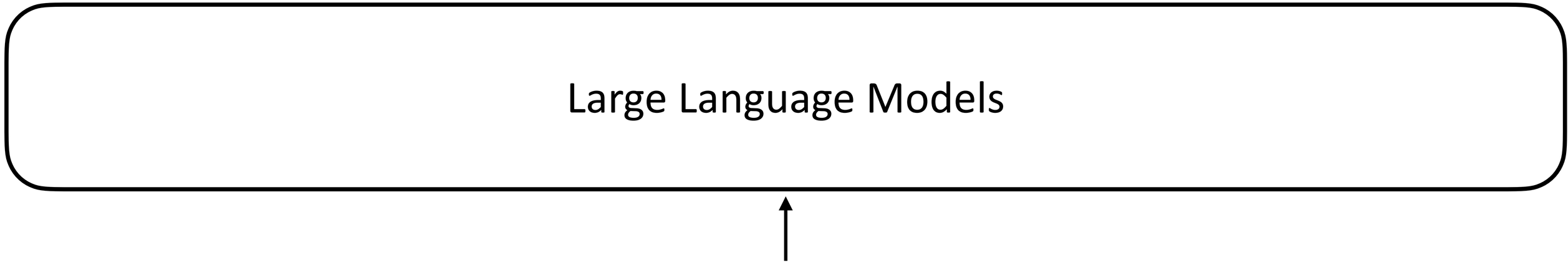

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No different from before, still shift one token left to obtain output

Review: Inference time for ChatBot

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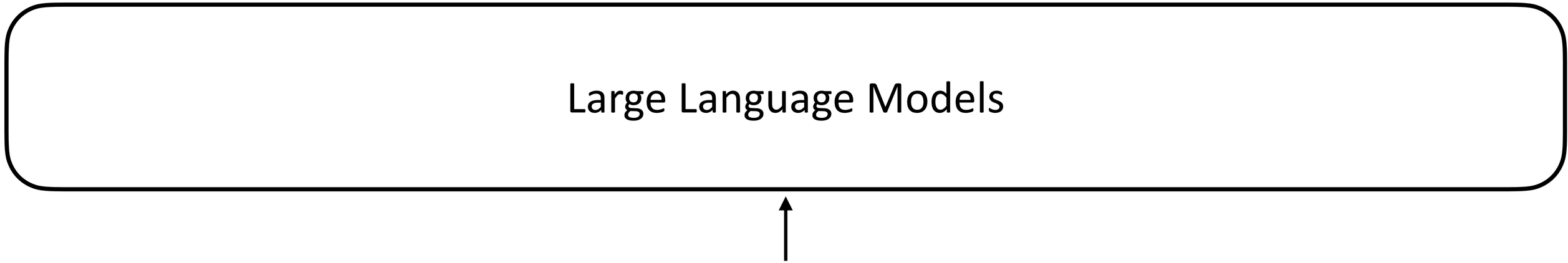

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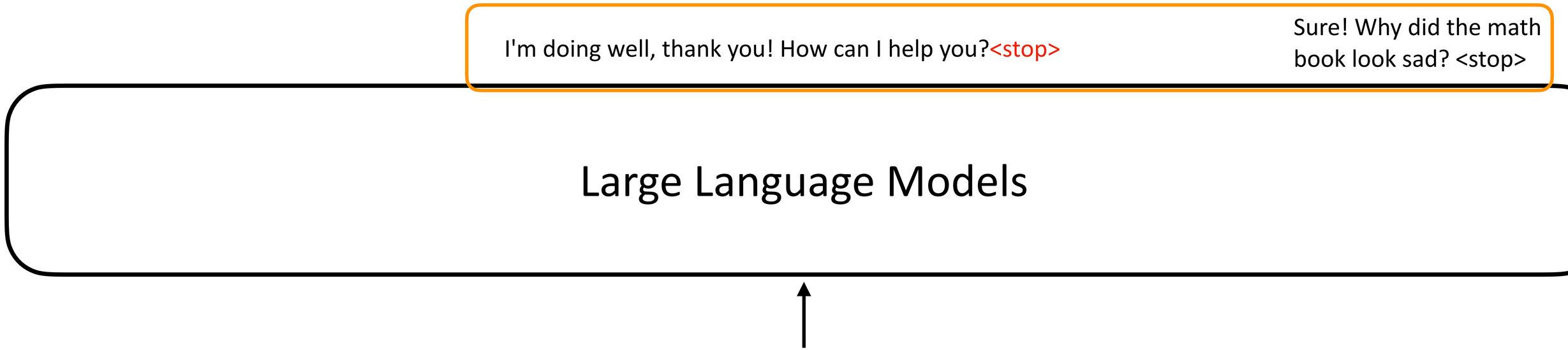

At inference time, we only ask the model to predict assistant parts

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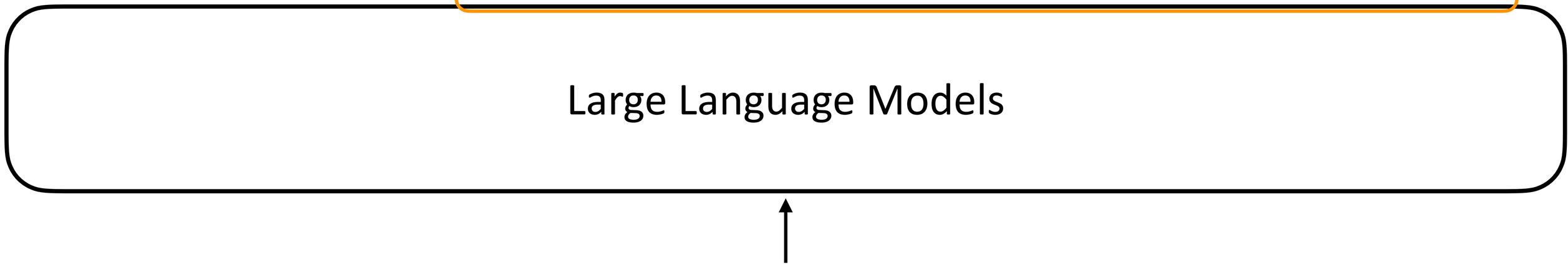

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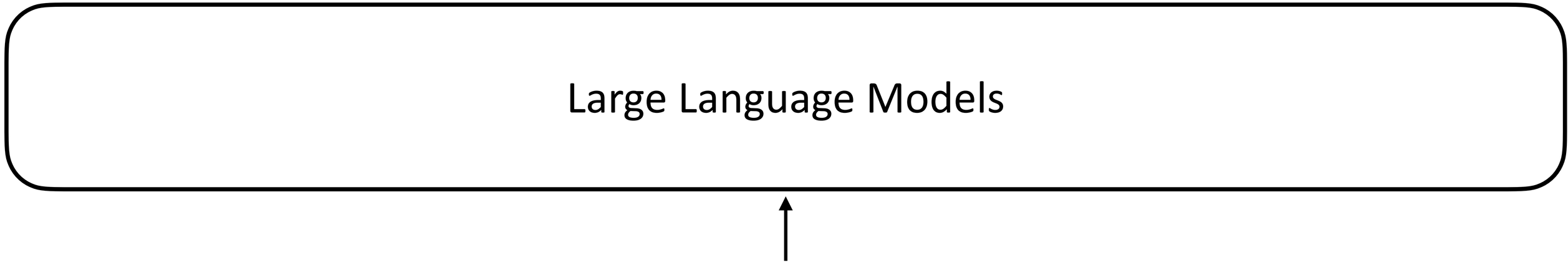
At inference time, we only ask the model to predict assistant parts

So typically, other parts are all masked when predicting the loss

Review: Inference time for ChatBot



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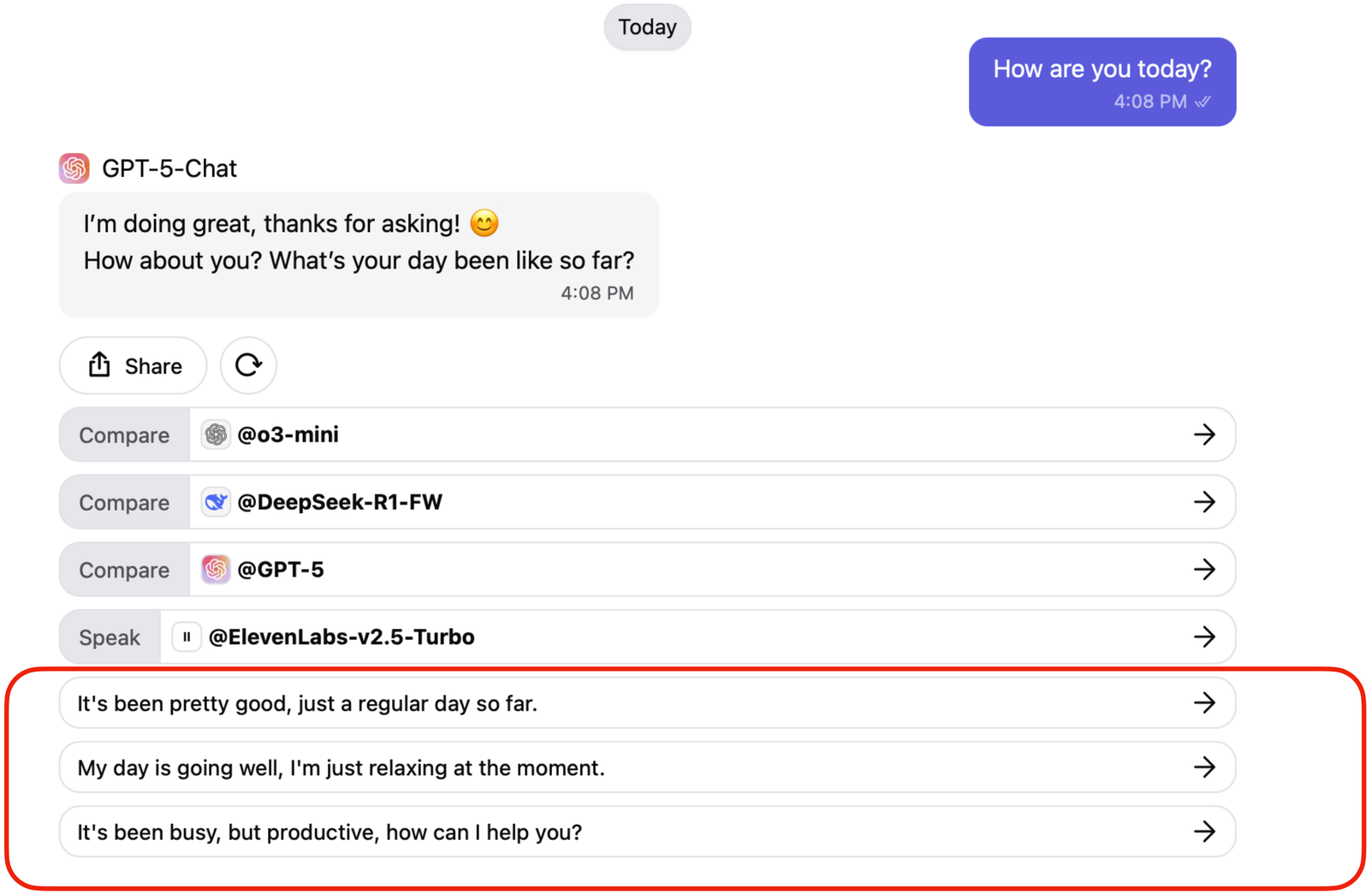
Or, we only mask <user> <assistant> tags, not learning all contents, then the chatbot can suggest questions each round

Review: Chatbot can ask questions if not masking



Reinforcement Learning

robot

Learning Tasks

- Supervised learning - $\mathcal{D} = \left\{\left(\boldsymbol{x}^{(i)}, y^{(i)}\right)\right\}_{i=1}^{N}$
  - Regression - $y^{(i)} \in \mathbb{R}$
  - Classification - $y^{(i)} \in \{1, \ldots, C\}$
- Unsupervised learning - $\mathcal{D} = \left\{\boldsymbol{x}^{(i)}\right\}_{i=1}^{N}$
  - Clustering
  - Dimensionality reduction

K-means

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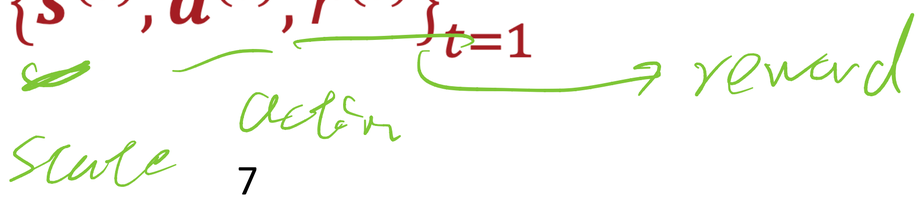
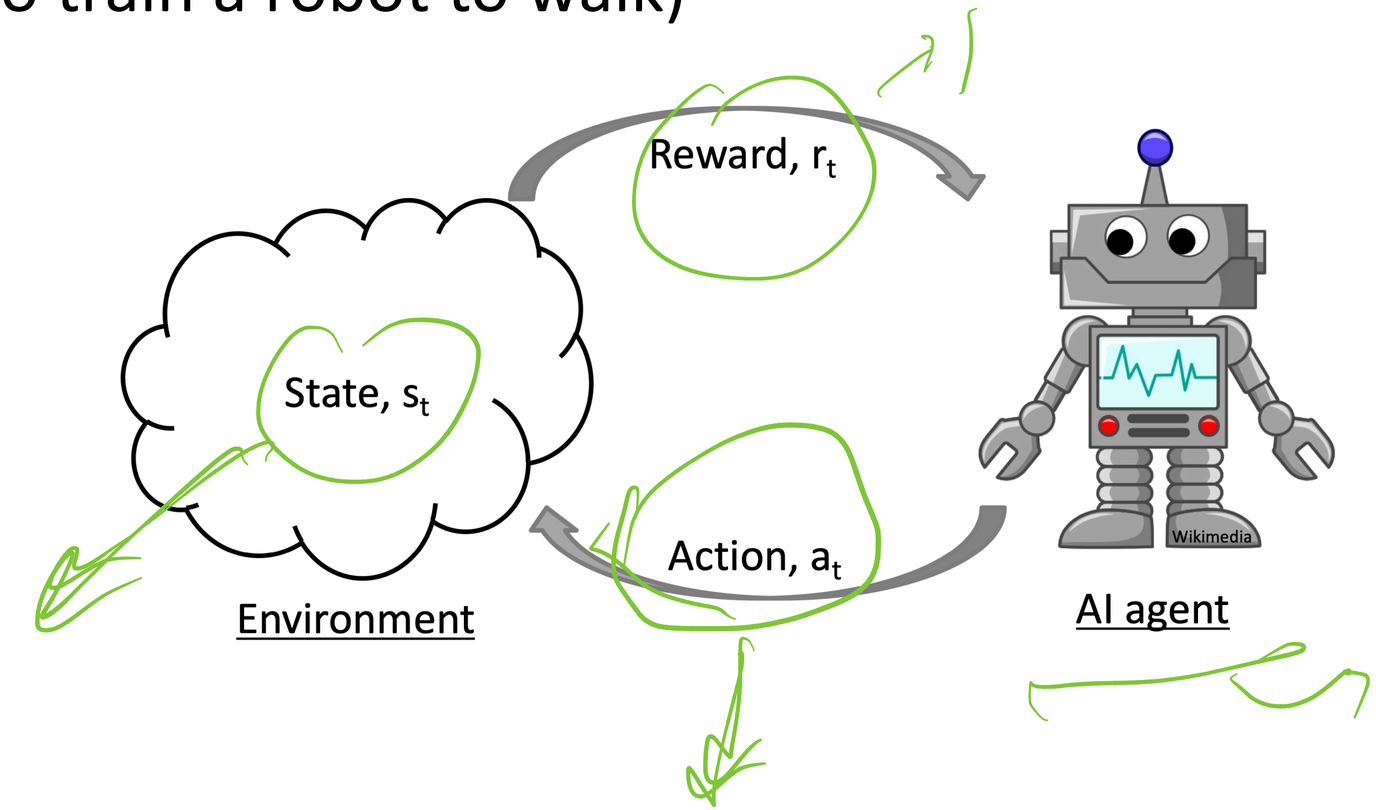
- Reinforcement learning - $\mathcal{D} = \left\{\boldsymbol{s}^{(t)}, \boldsymbol{a}^{(t)}, r^{(t)}\right\}_{t=1}^{T}$



RL Setup

In many cases, we cannot precisely define what the correct output is (think of we want to train a robot to walk)



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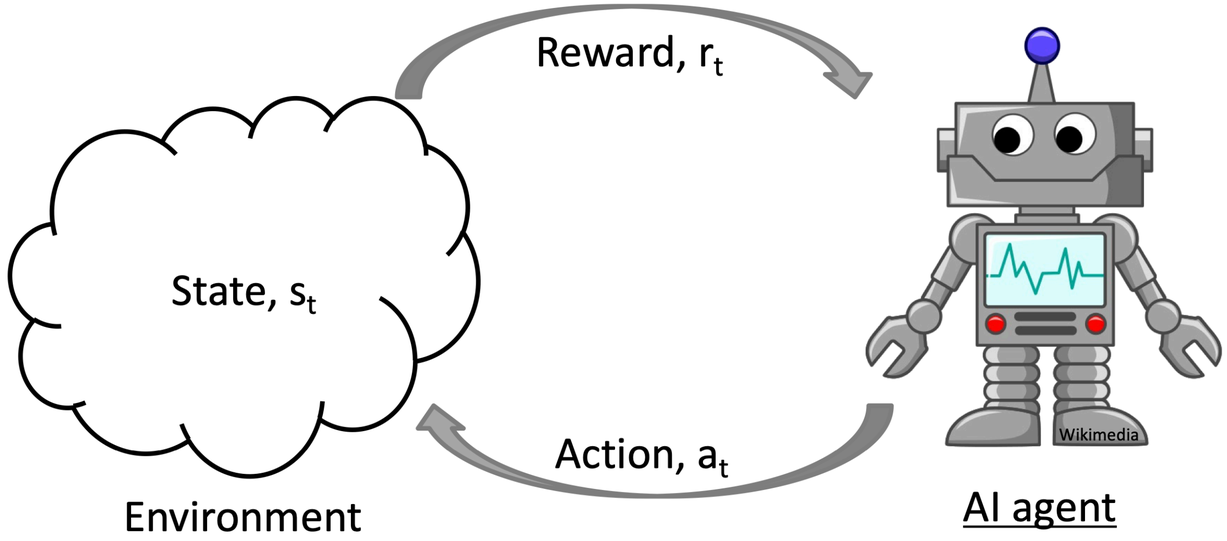

Agent chooses **actions** which can depend on past

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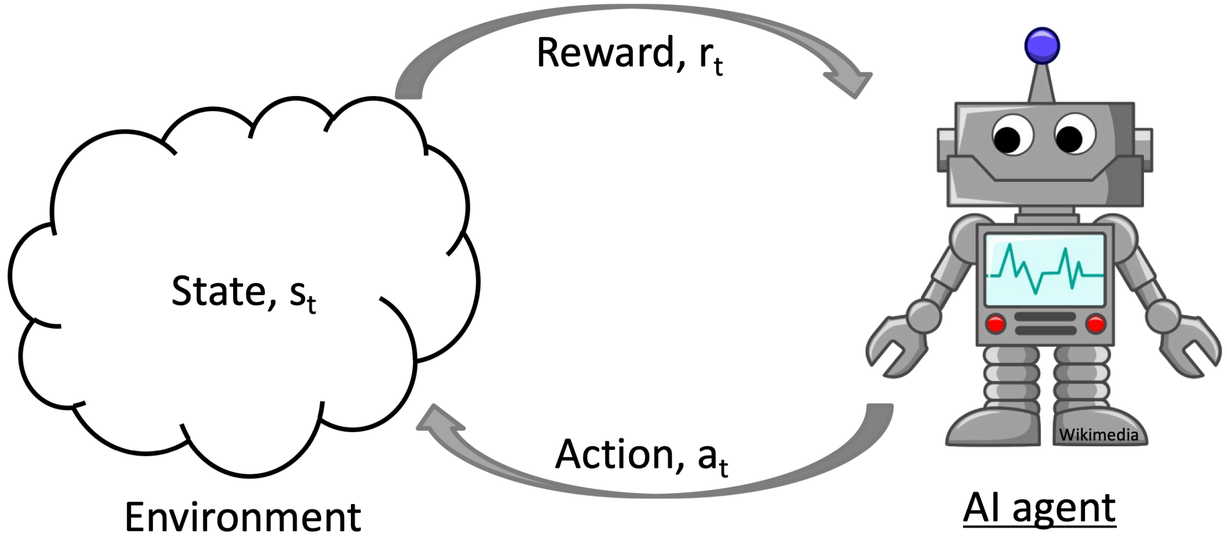

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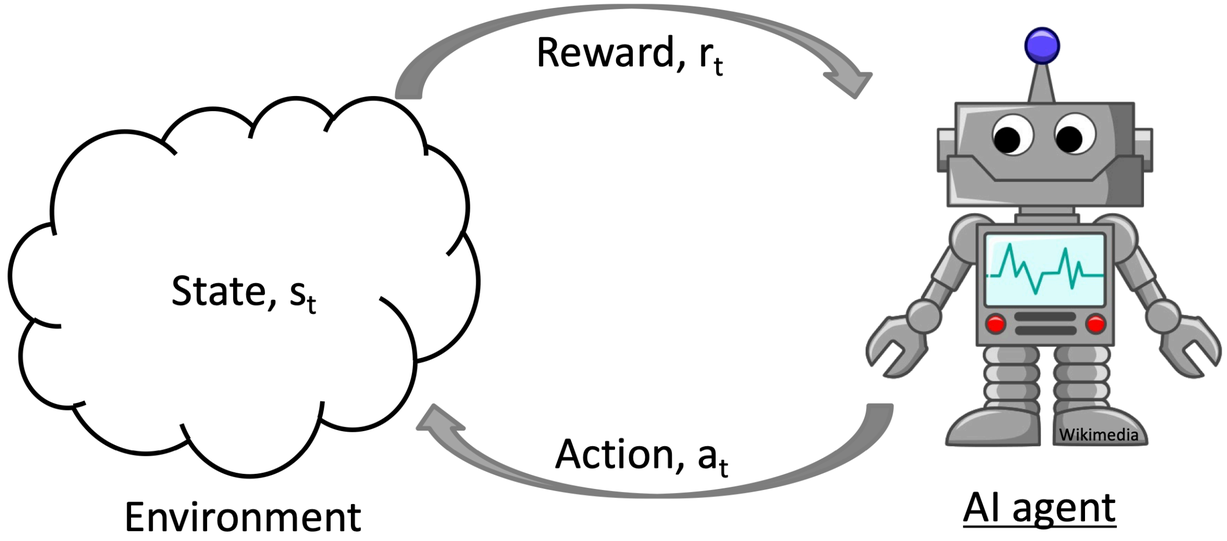

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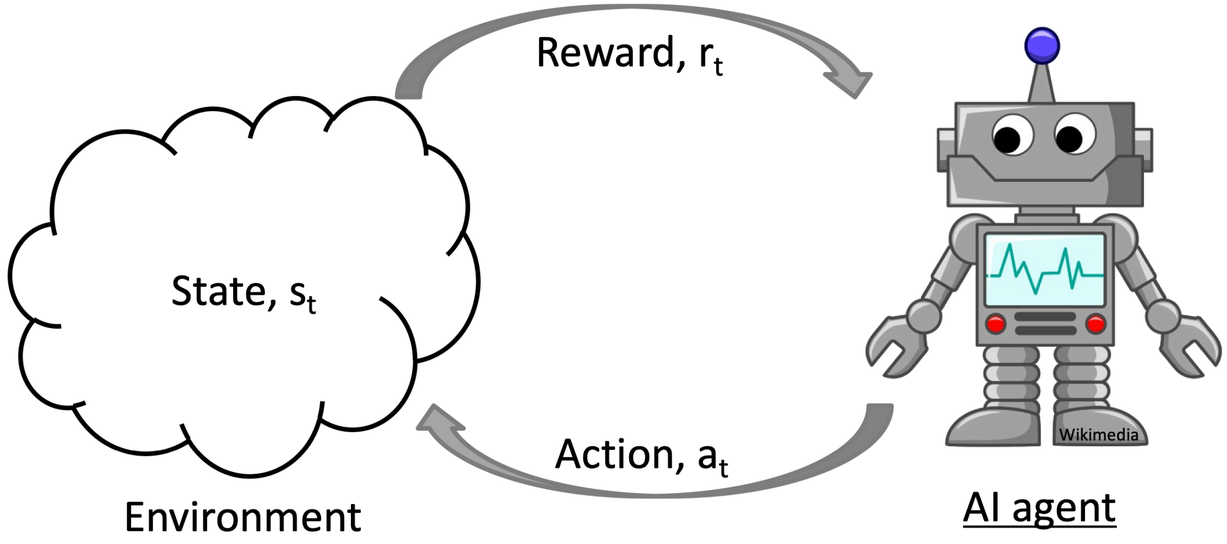

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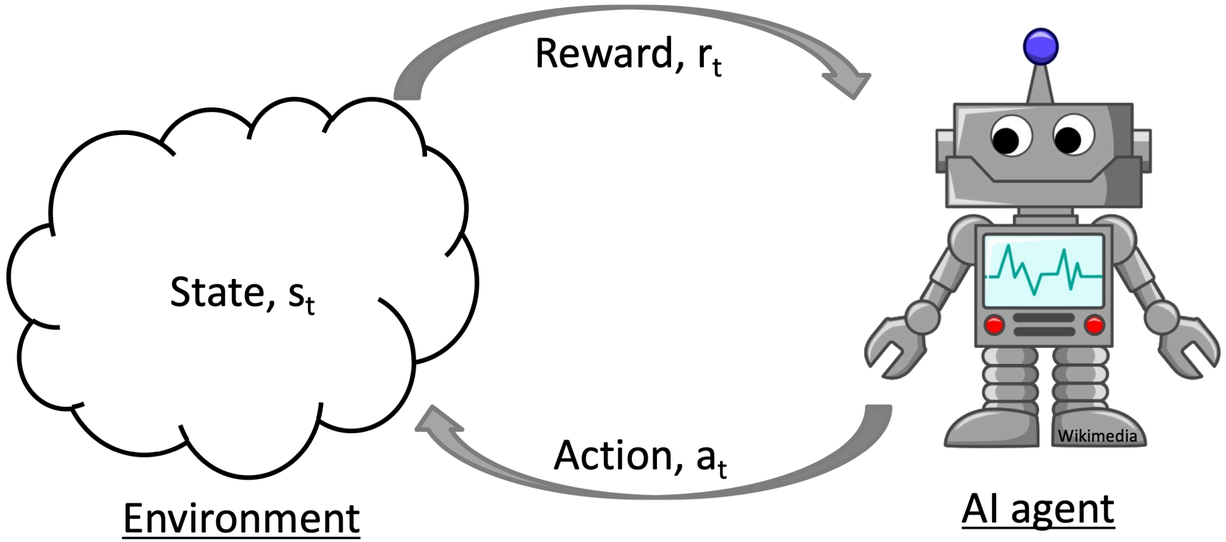
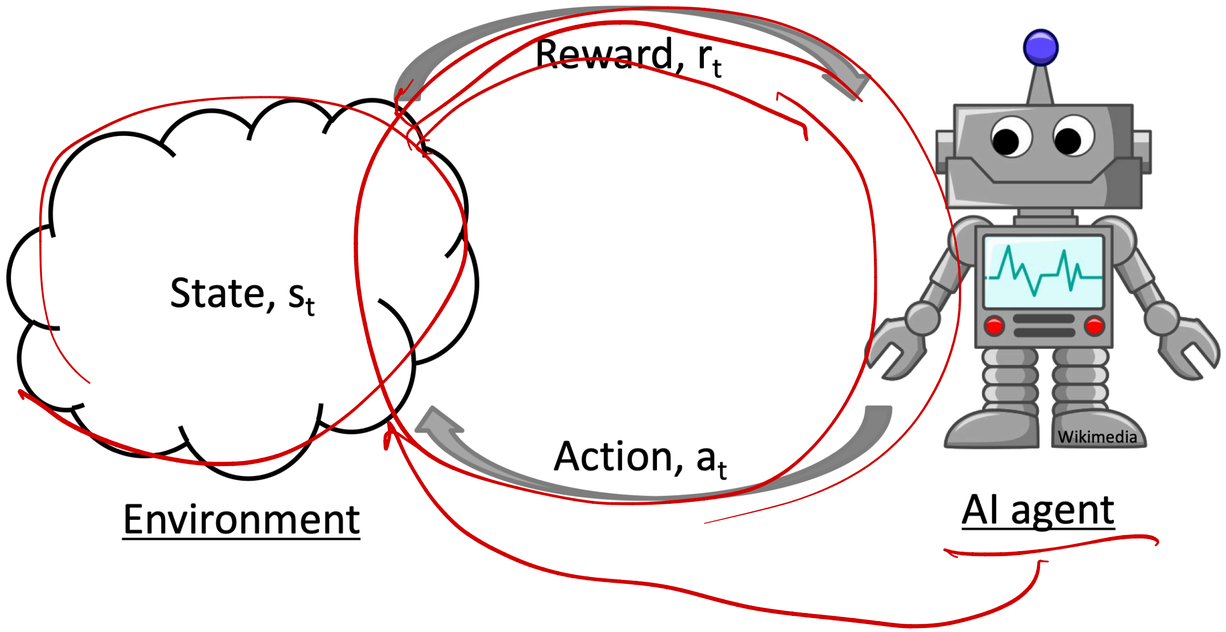
Reward (Output) depends on (Inputs) action and state of environment

Goal: maximize the total reward

Differences from Supervised Learning

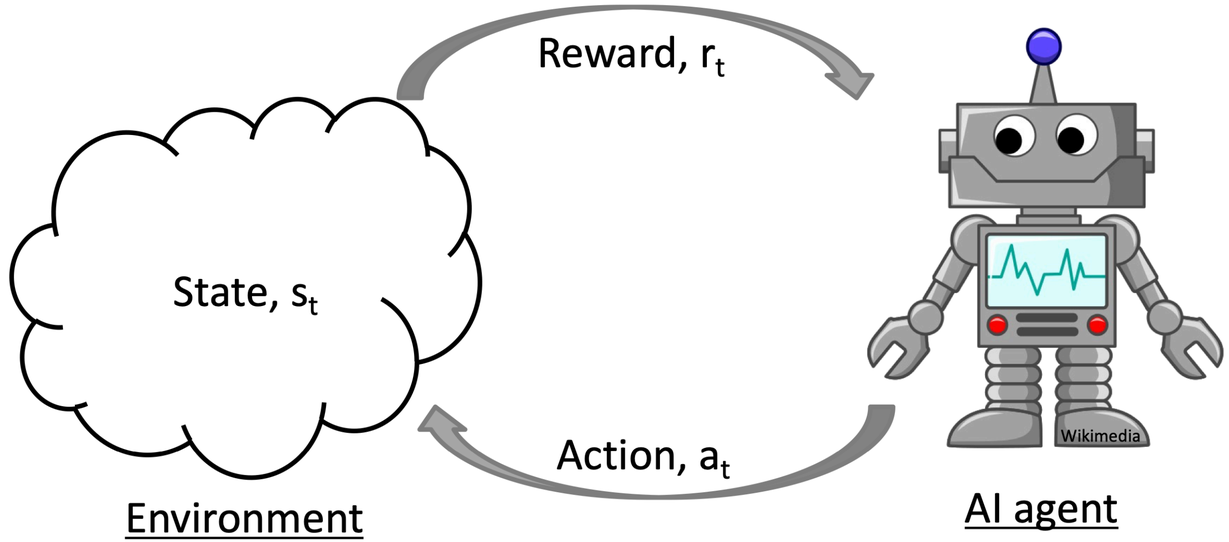


Differences from Supervised Learning



- Maximize reward (rather than learn reward)

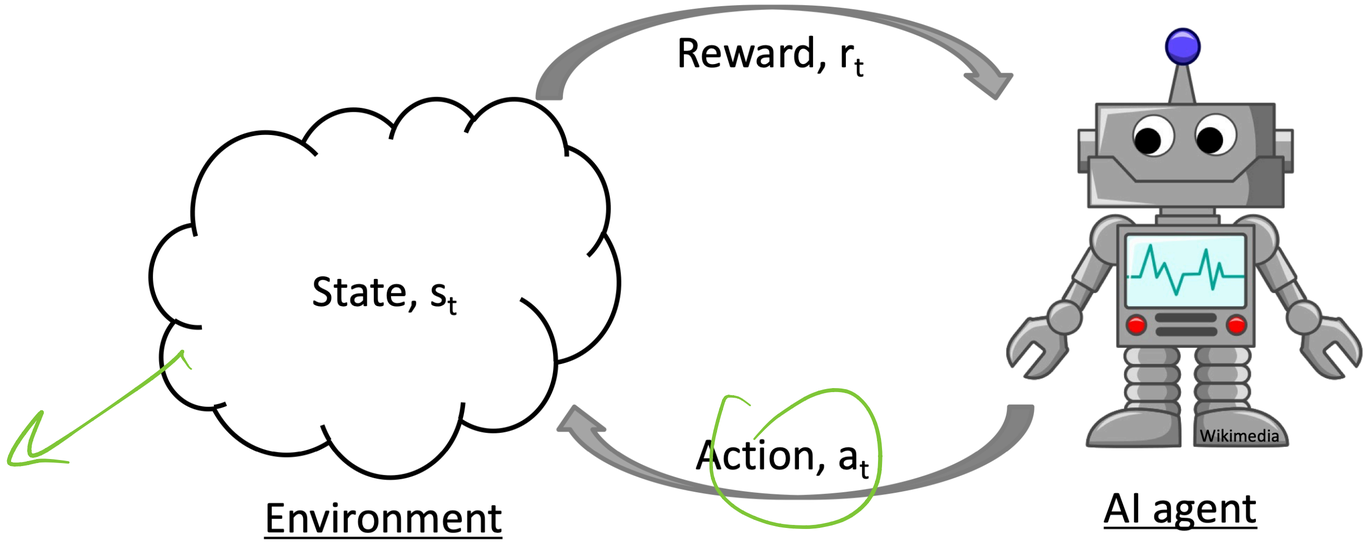
Differences from Supervised Learning



- Maximize reward (rather than learn reward)

Supervised training is like imitation

Differences from Supervised Learning

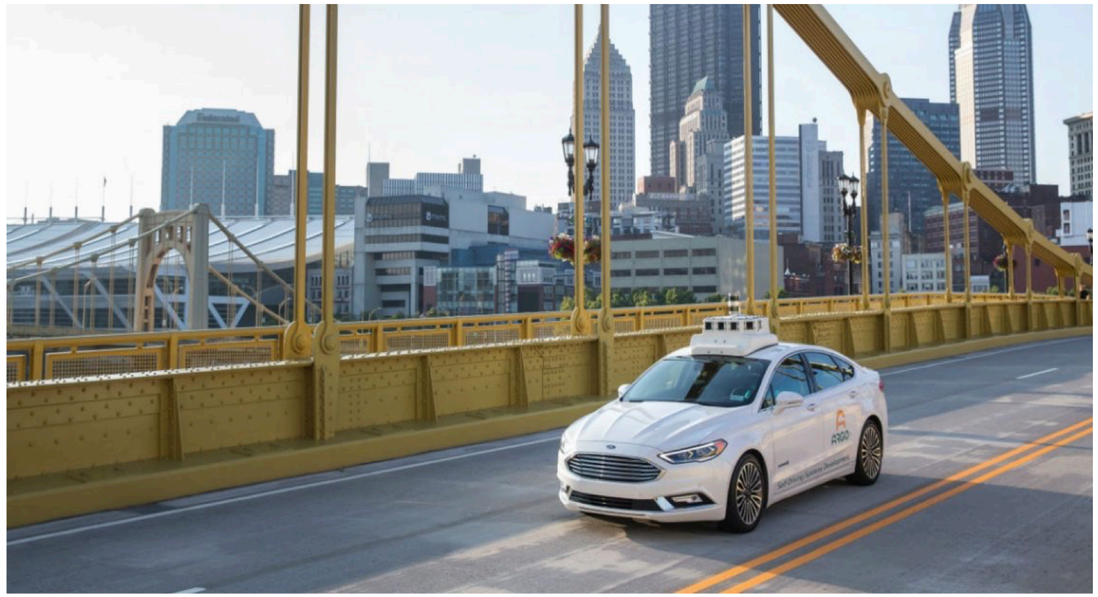


- Maximize reward (rather than learn reward) Supervised training is like imitation
- Inputs are not iid – state & action depends on past

RL Examples

(x, y)

imitation learning

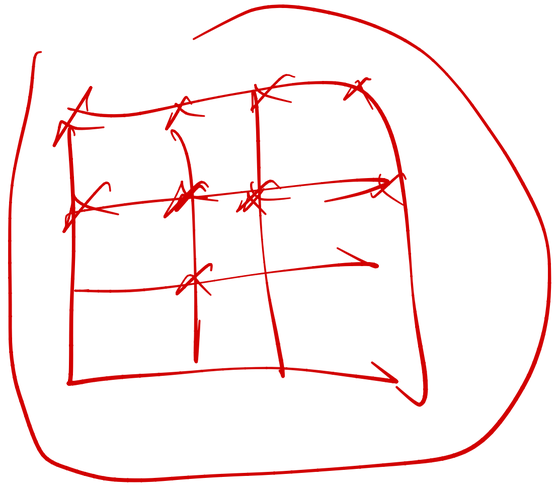


Reward

RL Setup

RL Setup

- State space, $\mathcal{S}$
- Action space, $\mathcal{A}$



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RL Setup

- State space, $\mathcal{S}$
- Action space, $\mathcal{A}$
- Reward function
  - Stochastic, $p(r \mid s, a)$
  - Deterministic, $R\colon \mathcal{S} \times \mathcal{A} \to \mathbb{R}$

S,

R (S, A) ⟶ Value

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- Transition function
  - Stochastic, $p(s' \mid s, a)$
  - Deterministic, $\delta\colon \mathcal{S} \times \mathcal{A} \to \mathcal{S}$

transition $(S, a) \to S$

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- Reward and transition functions can be known or unknown

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In this lecture, we assume they are known

- Reward and transition functions can be known or unknown

RL Setup

- Policy, $\pi : \mathcal{S} \rightarrow \mathcal{A}$ $\longrightarrow a = \pi(s)$.
  - Specifies an action to take in *every* state

RL Setup

- Policy, $\pi : \mathcal{S} \rightarrow \mathcal{A}$
  - Specifies an action to take in *every* state

- Value function, $V^{\pi}: \mathcal{S} \rightarrow \mathbb{R}$
  - Measures the expected total reward of starting in some state s and executing policy π, i.e., in every state, taking the action that π returns

RL Example - gridworld

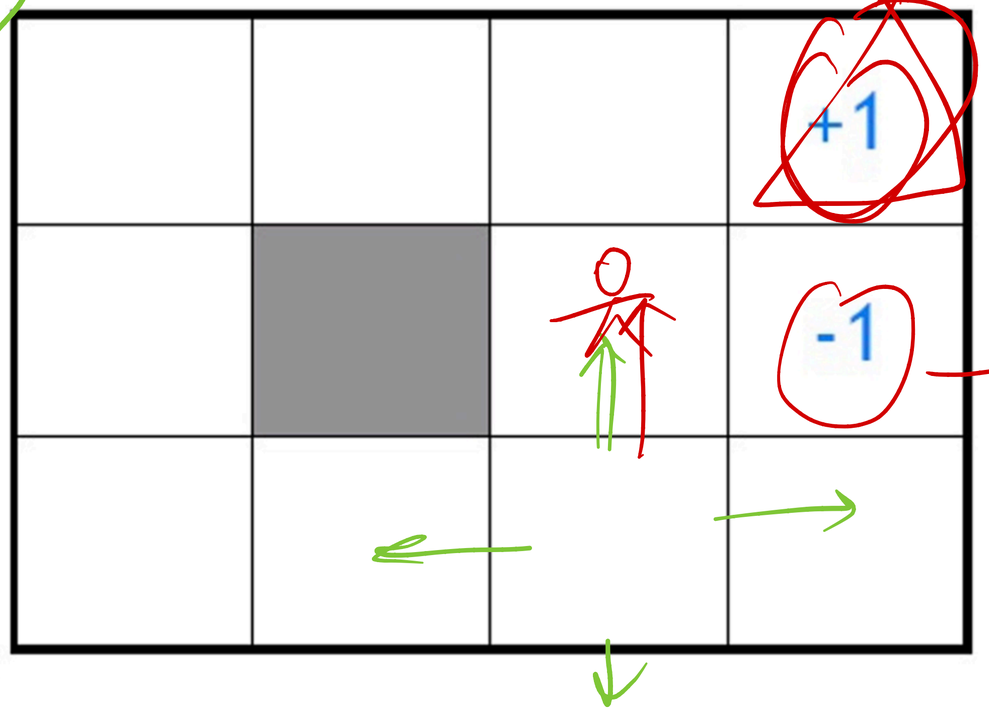
$\mathcal{S}$ = all empty squares in the grid

$\mathcal{A}$ = {up, down, left, right}

Deterministic transitions

Rewards of +1 and -1 for entering the labelled squares

Terminate after receiving either reward



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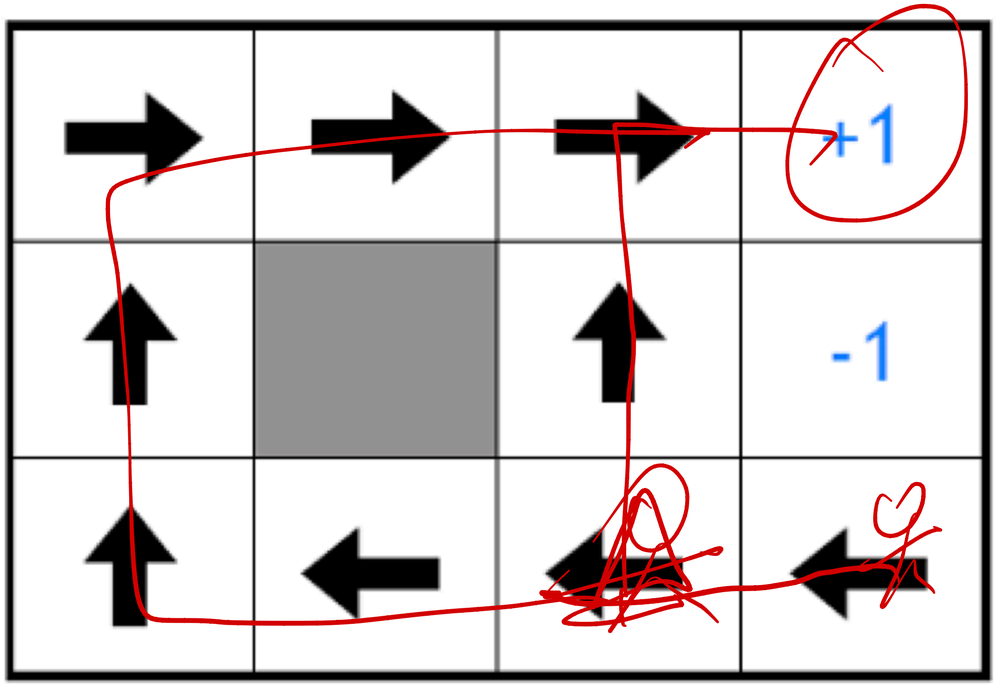

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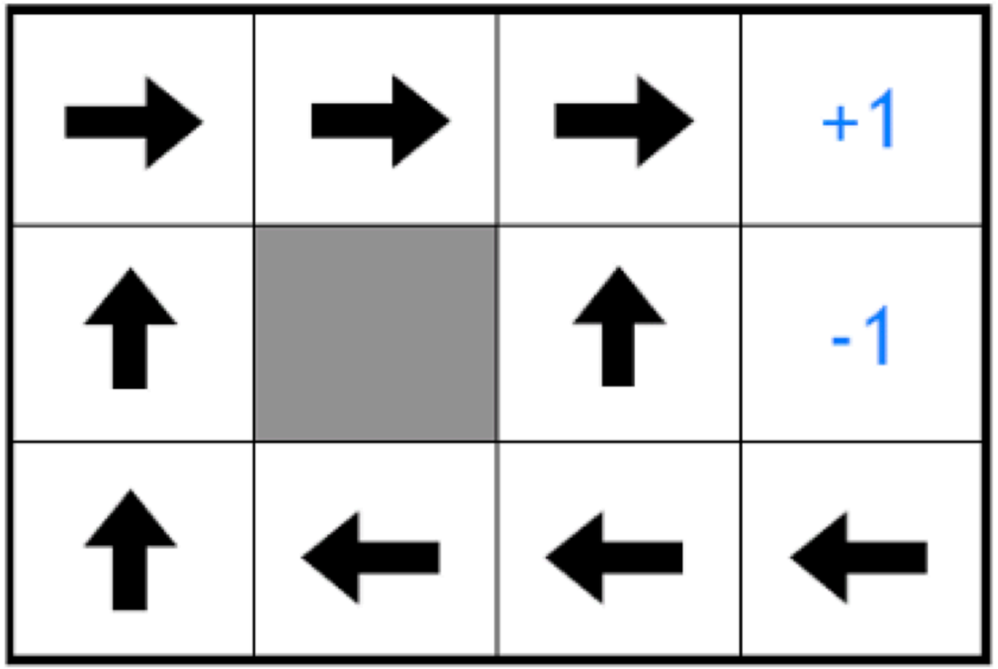


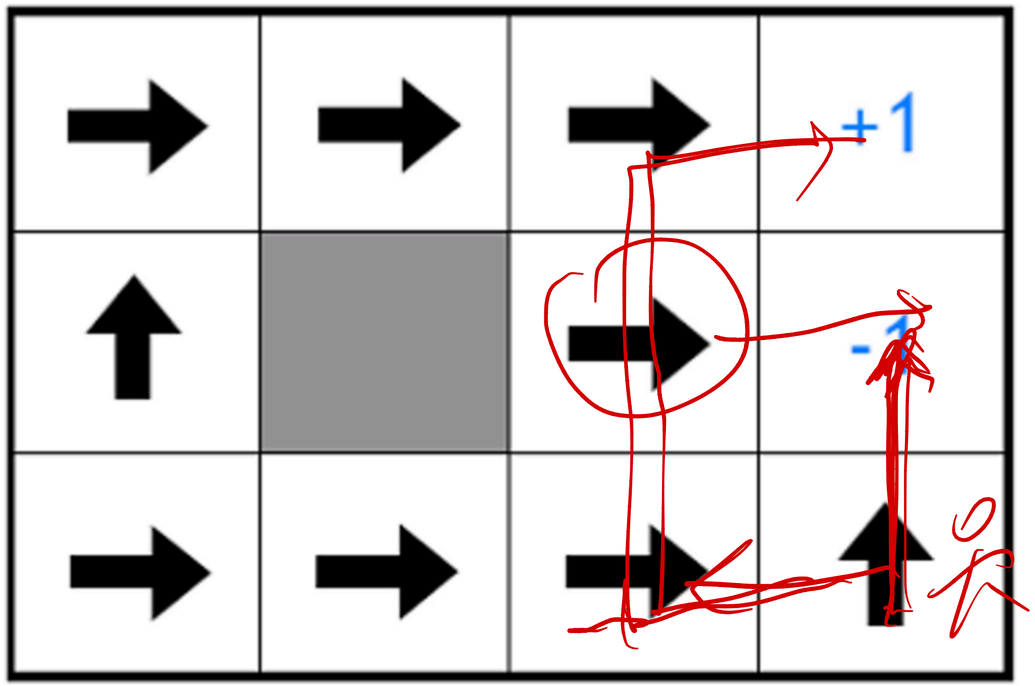
Is this policy optimal?

RL Example - gridworld

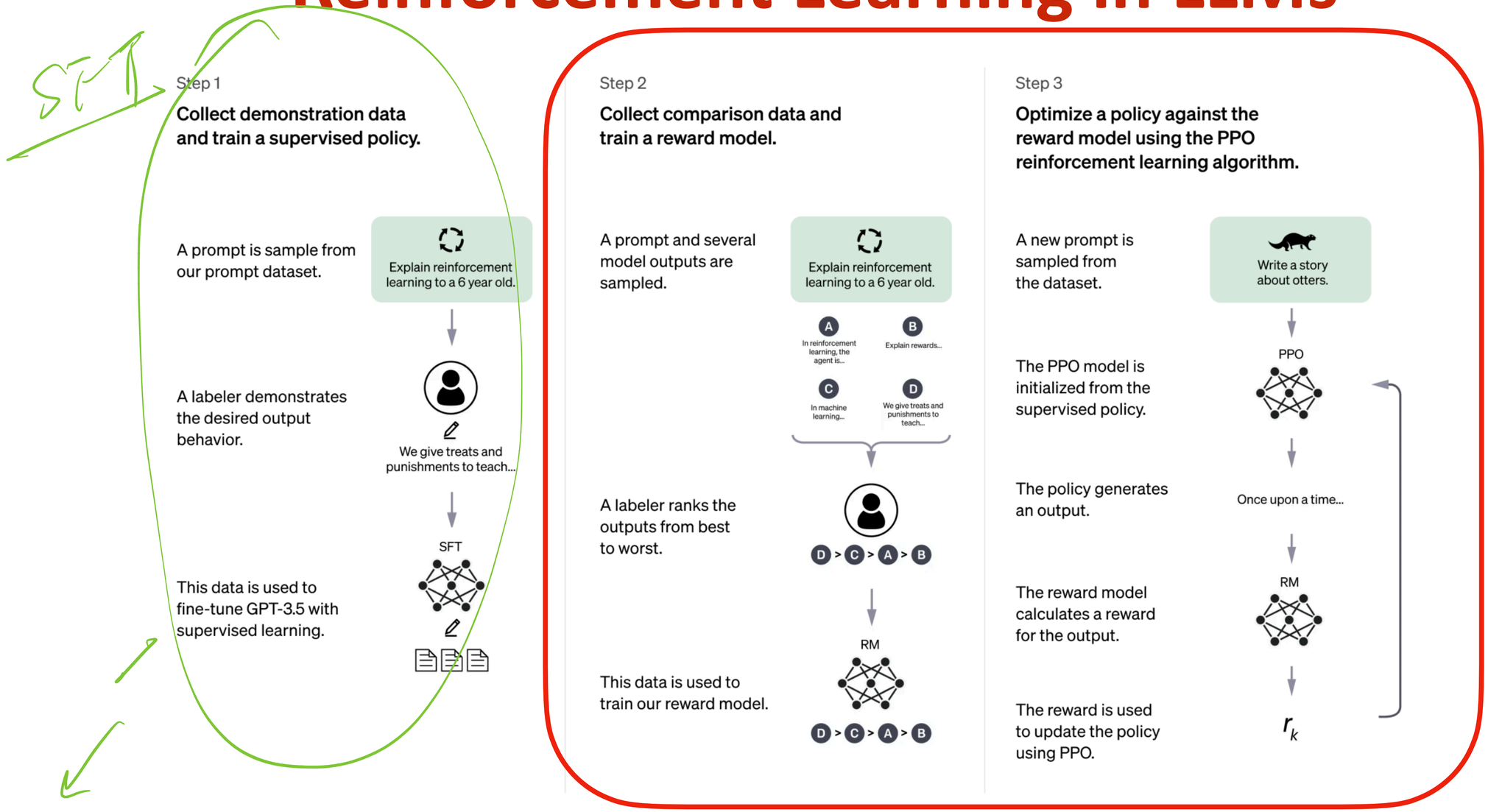
Optimal policy given a reward of -2 per step

RL Example - gridworld

Optimal policy given a reward of -2 per step



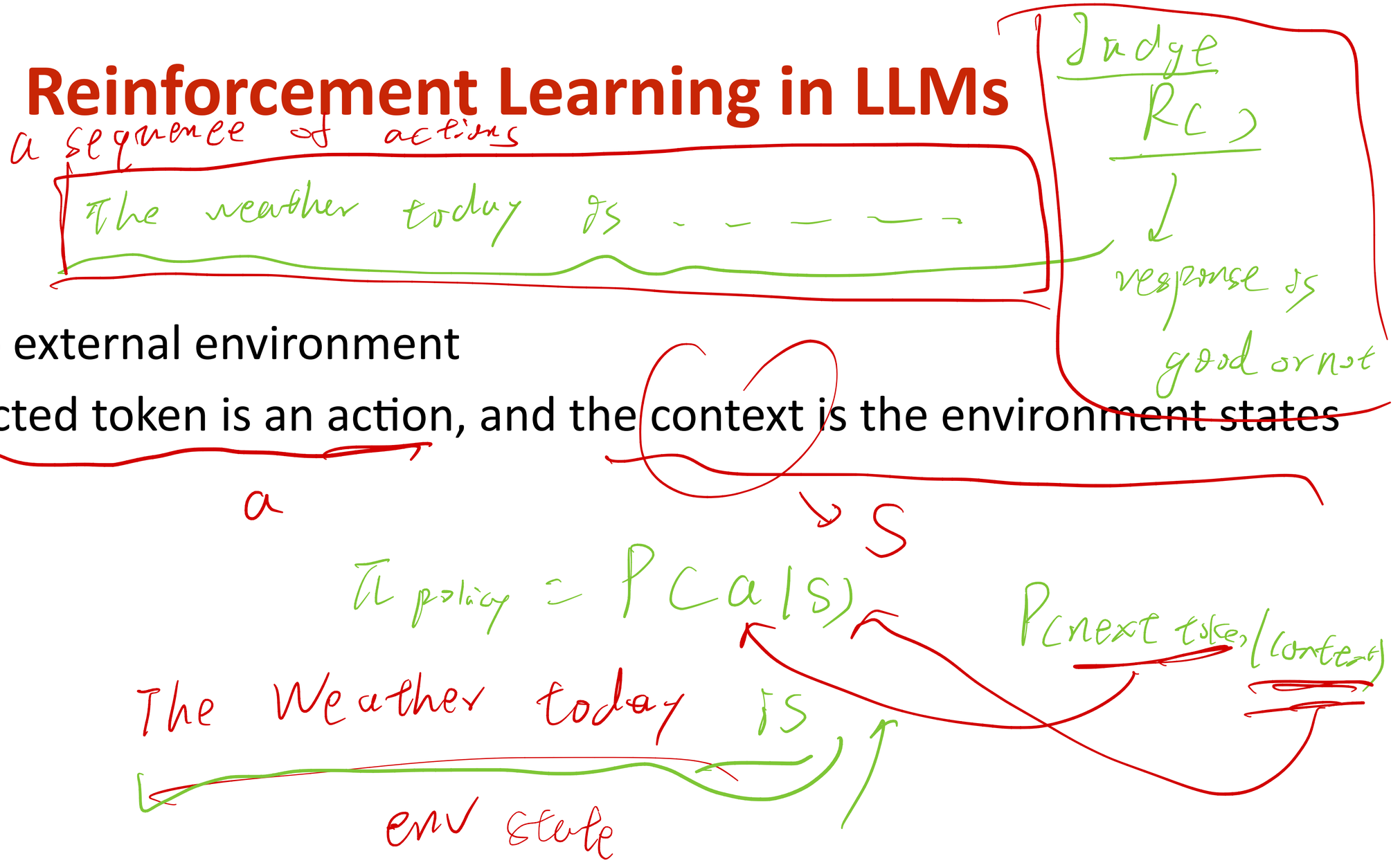
Reinforcement Learning in LLMs



SFT

chatGPT

Reinforcement Learning in LLMs



From Imitation to Optimization

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Imitation (SFT)

Fit $\hat{p}(y|x) \approx p^*(y|x)$ for some reference distribution $p^*(y|x)$

- Pure generative modeling perspective
- Requires samples from reference policy

From Imitation to Optimization

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Optimization (RLHF)

Find $\hat{p}(y|x)$ such that $\max_p E_p[R(y,x)]$ for a reward $R(y,x)$

- Maximize some reward function that we can measure
- LMs are policies, not a model of some distribution

Reward Optimization in Language Models

Reward Optimization in Language Models

$p_\theta(x)$ Language Model

θ: parameter

Objective:

$$\theta = \arg\max_{\theta} \mathbb{E}_{x \sim p_\theta(x)} R(x)$$

Reward Optimization in Language Models

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X is the language sequence, R is the reward function, scores the generated response (we can consider R as a human or a model)

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How to do gradients over this objective?

$R(\cdot)$ is not related to θ

θ:

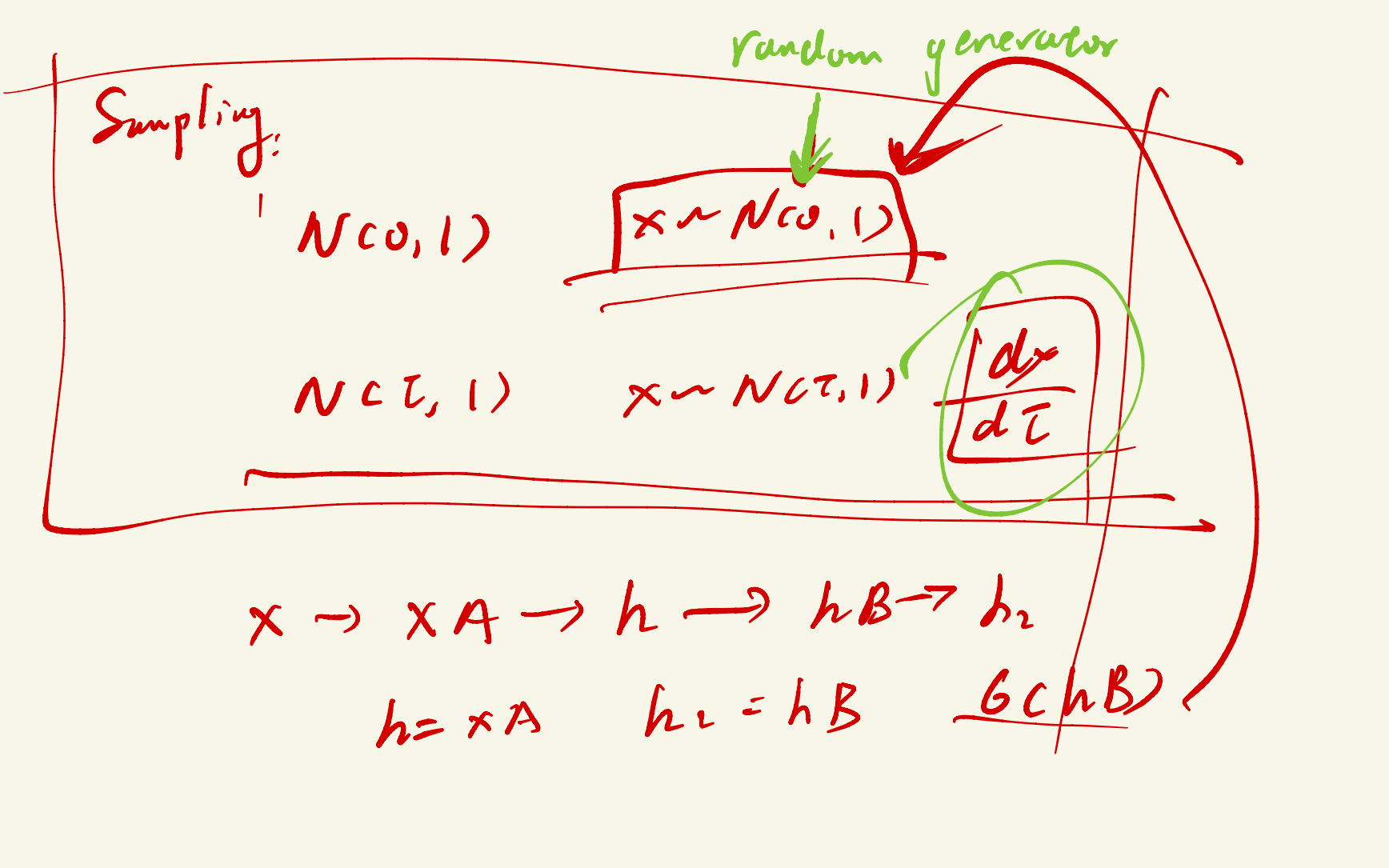
Sampling is often ✗ language

$x_1, x_2, x_3 \sim p_\theta(x)$

differentiable

$$\frac{R(x_1) + R(x_2) + R(x_3)}{3}$$

X is discrete


random generator
Sampling:
N(0,1)
x ~ N(0,1)
N(τ,1)
x ~ N(τ,1)
dx/dτ
x → xA → h → hB → h₂
h = xA
h₂ = hB
G(hB)

Reward Optimization in Language Models

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Gradient estimation (policy gradient):

$$\nabla_\theta \mathbb{E}_{x \sim p_\theta(x)} R(x)$$

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Gradient estimation (policy gradient):

$$\hat{g} = \mathbb{E}_{x \sim p_\theta(x)} R(x) \nabla_\theta \log p_\theta(x)$$

$\hat{g}$ is the gradient (not objective)

Policy Gradient:

Objective: $\bar{E}_{x \sim P_\theta(x)} R(x)$

$$\nabla_\theta \bar{E}_{x \sim P_\theta(x)} R(x) = \nabla_\theta \sum_x P_\theta(x) R(x)$$

$$= \sum_x \nabla_\theta P_\theta(x) R(x)$$

$\nabla_\theta P_\theta(x) = P_\theta(x) \cdot \nabla_\theta \log P_\theta(x)$

$\nabla_\theta \log P_\theta(x) = \dfrac{\nabla_\theta P_\theta(x)}{P_\theta}$

$$= \sum_x P_\theta(x) \nabla_\theta \log P_\theta(x) R(x)$$

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REINFORCE / Policy Gradient

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How to implement?

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How to implement?

batch

Objective = $\sum_{i=1}^{n} \frac{1}{n} R(x^{(i)}) \log p_\theta(x^{(i)})$

$x^{(1)}, \ldots, x^{(n)} \sim p_\theta(x)$

≠ objective = $\hat{g}$

no gradient

REINFORCE / Policy Gradient

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Objective = $\sum_{i=1}^{n} \frac{1}{n} R(x^{(i)}) \log p_\theta(x^{(i)})$ $\quad x^{(1)}, \ldots, x^{(n)} \sim p_\theta(x)$

SFT: $\sum_{i=1}^{n} \frac{1}{n} \log p_\theta(x^{(i)})$

$x^{(i)} \sim$ dataset

This objective looks kinda like ~~weighted~~ log likelihood maximization?

REINFORCE / Policy Gradient

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1. Have a weight of R(x)

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2. The data x is sampled from the model itself, not from a static dataset

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This equation is not that complex, just view it as a weighted likelihood maximization

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This is the simplest form of RL, many other RL algorithms (PPO, GRPO) are more like variants of this simple equation with the same spirit

REINFORCE / Policy Gradient

$$\text{Objective} = \sum_{i=1}^{n} \frac{1}{n} R(x^{(i)}) \log p_\theta(x^{(i)})$$

$$x^{(1)}, \ldots, x^{(n)} \sim p_\theta(x)$$

1. Have a weight of R(x)
2. The data x is sampled from the model itself, not from a static dataset

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This is the simplest form of RL, many other RL algorithms (PPO, GRPO) are more like variants of this simple equation with the same spirit

We can see why RL is called "self-improving", and it is trained by "synthetic data"

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1. Sample data from the model (or we call policy) itself (exploration)

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1. Sample data from the model (or we call policy) itself (exploration)
2. A reward function judges whether the explored data is good or bad

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Reinforcement learning is a mixed art of both training and inference during training time

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Why?

REINFORCE / Policy Gradient

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Training

REINFORCE / Policy Gradient

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Training

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Inference

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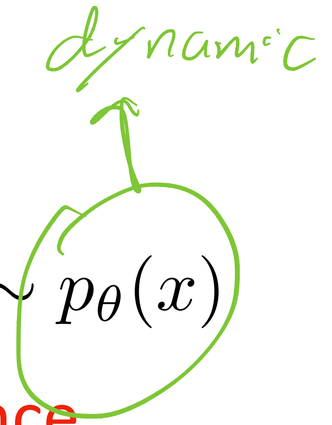
Inference



Each training step, the algorithm needs to run inference again

REINFORCE / Policy Gradient

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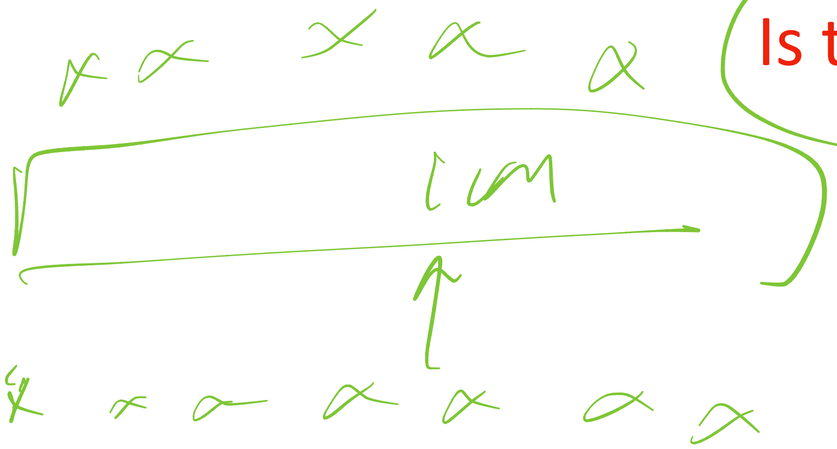
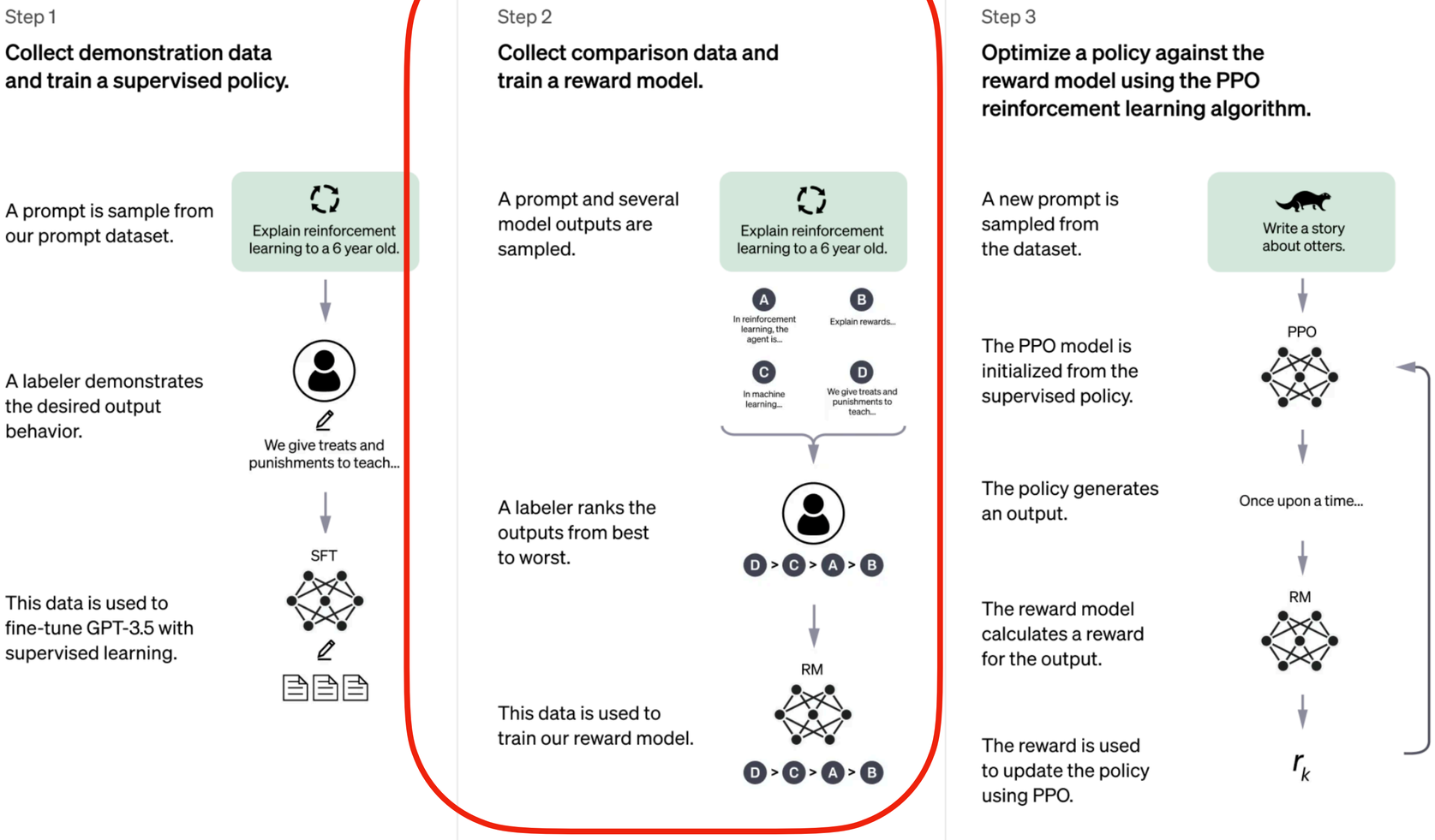
Is this efficient?



REINFORCE / Policy Gradient for Language Models

Objective = $$\sum_{i=1}^{n} \frac{1}{n} R(\{x_1^{(i)}, x_2^{(i)}, x_t^{(i)}\} \sum_{j=1}^{t} \log p_\theta(x_j^{(i)} | x_{<j}^{(i)})$$

Training a Reward Model in RLHF



Training a Reward Model

Suppose we have K responses and have them ranked by humans, then for all possible pairs of responses, y_w is the preferred one, y_l is the less preferred one, the objective of reward model $r_\theta(x, y)$ is:

$$\text{Objective} = -\frac{1}{\binom{K}{2}} E_{(x,y_w,y_l)\sim D} \left[\log\left(\sigma\left(r_\theta\left(x, y_w\right) - r_\theta\left(x, y_l\right)\right)\right)\right]$$

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In practice, we can just sample a subset of pairs, rather than enumerating all pairs

Reinforcement Learning Relaxes Human Annotations

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	Base model →	Supervised learning (one time) →	Pairwise feedback →	RL →	Evaluation
Compute cost	$300k	$100	$100	$100	$0
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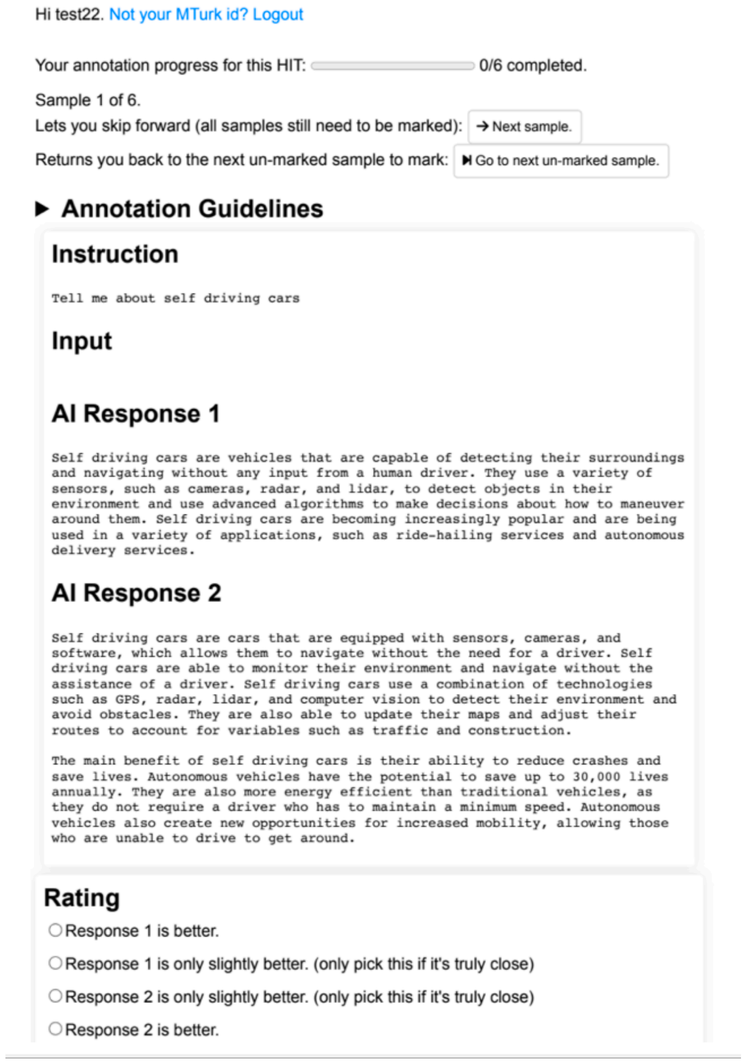
1. SFT data can be expensive
2. There may be tasks that are much easier for humans to verify than solve, particularly when AI today is outperforming humans

Reinforcement Learning from Human Feedback Data

Basic Setup



Reinforcement Learning from Human Feedback Data

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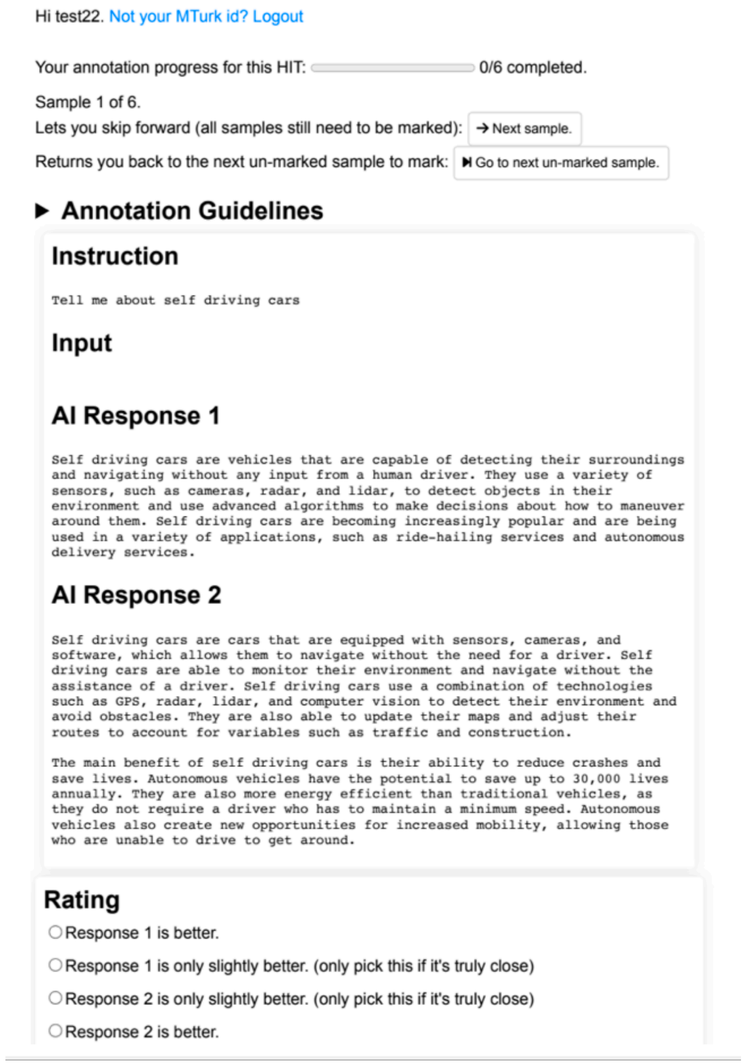

Easier than annotating the responses directly

Thank You!