

Reinforcement Learning Basics

Junxian He

Oct 15, 2025

Peason J S R L

Review: Multi-Turn Instruction Tuning

<User> How are you today?\n<Assistant> I'm doing well, thank you! How can I help you? \n\n<User> Can you tell me a joke? \n<Assistant> Sure! Why did the math book look sad? <stop>

Large Language Models

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Why did the math book look sad?

No different from before, still shift one token left to obtain output

I'm doing well, thank you! How can I help you?<stop>

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Large Language Models

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Large Language Models

At inference time, we only ask the model to predict assistant parts

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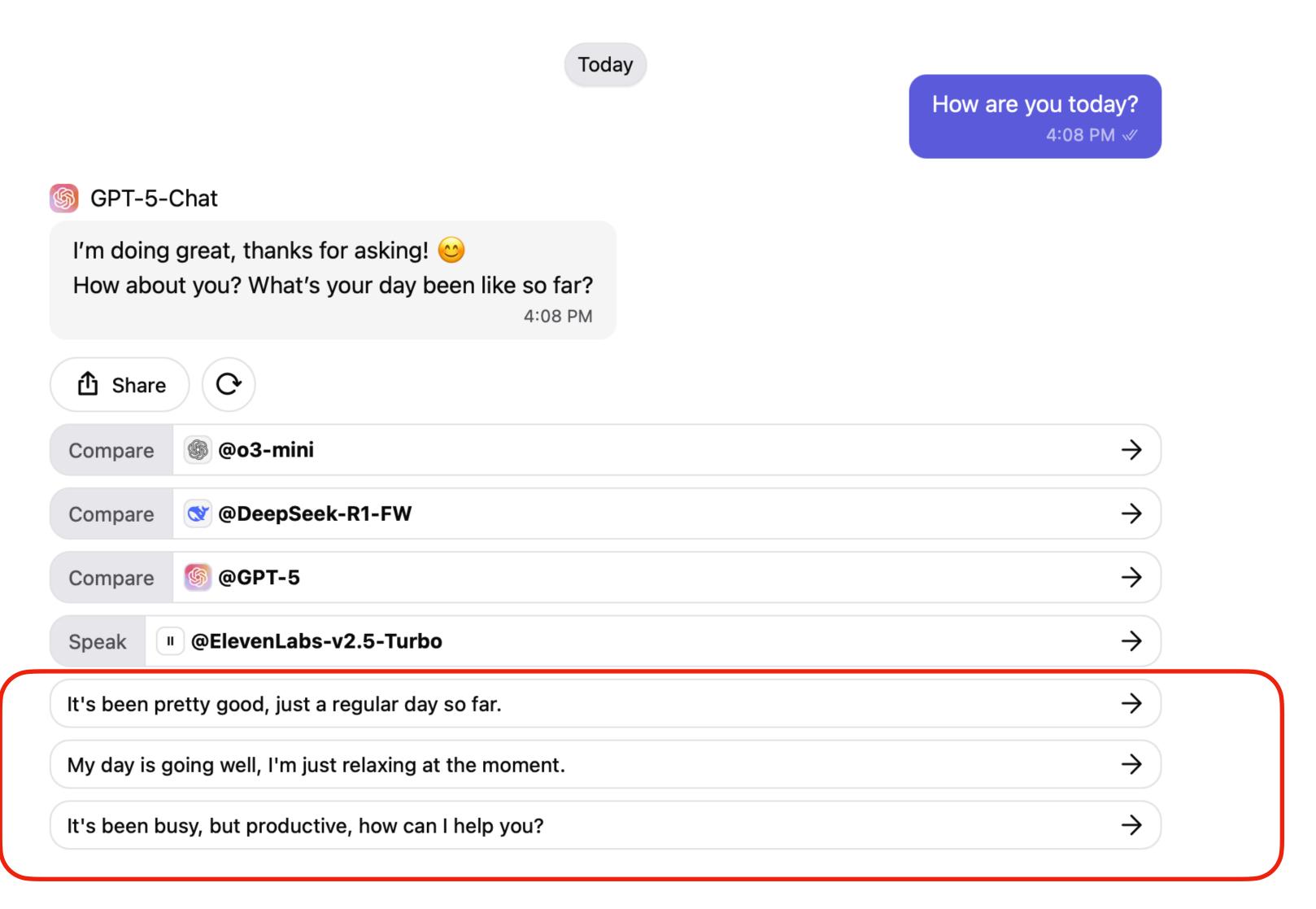
Large Language Models

At inference time, we only ask the model to predict assistant parts. So typically, other parts are all masked when predicting the loss

Large Language Models

Or, we only mask <user> <assistant> tags, not learning all contents, then the chatbot can suggest questions each round

Review: Chatbot can ask questions if not masking



Reinforcement Learning



Learning Tasks

- Supervised learning $\mathcal{D} = \{(x^{(i)}, y^{(i)})\}_{i=1}^N$

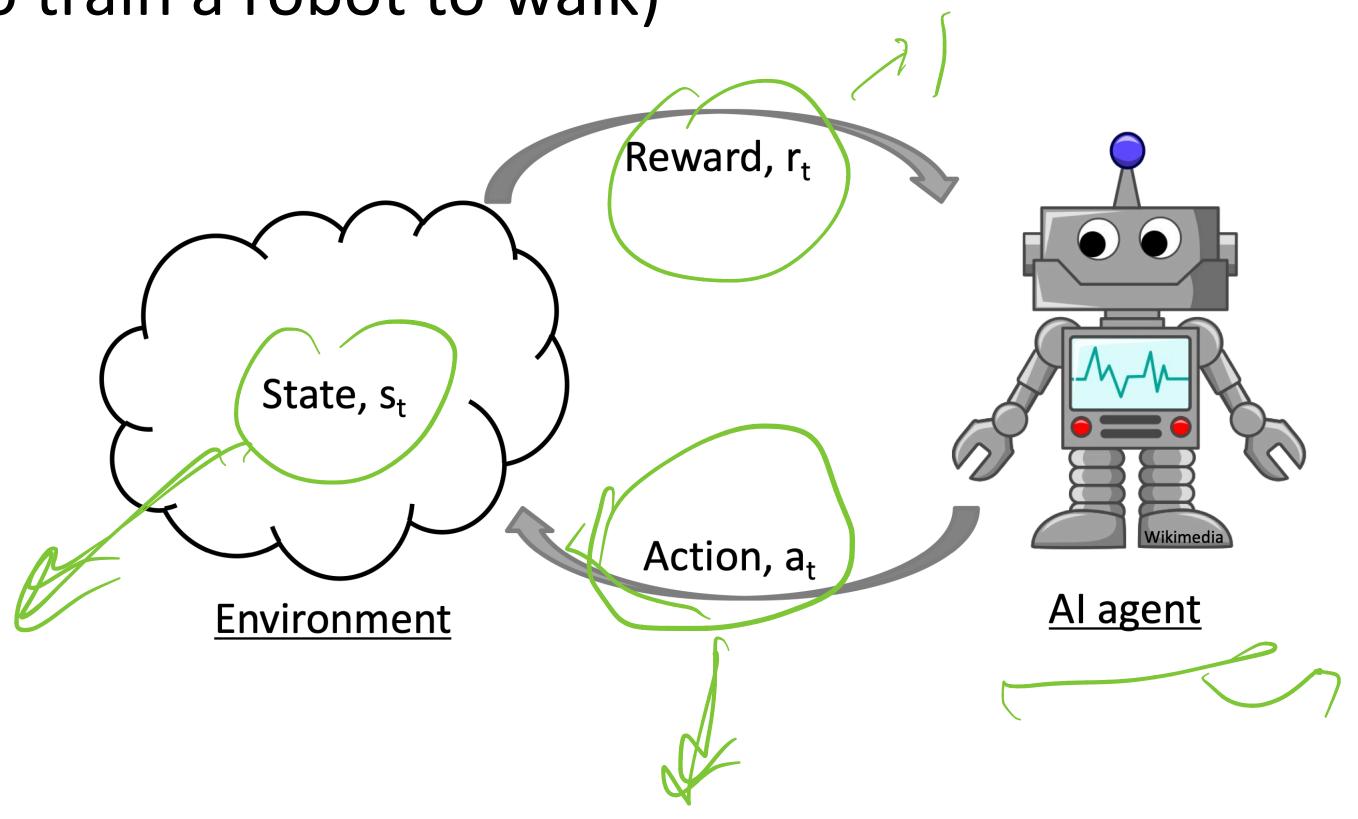
 - Regression $y^{(i)} \in \mathbb{R}$ Classification $y^{(i)} \in \{1, ..., C\}$
- Unsupervised learning $\mathcal{D} = \{x^{(i)}\}_{i=1}^{N}$
 - Clustering
 - Dimensionality reduction

K-me ans

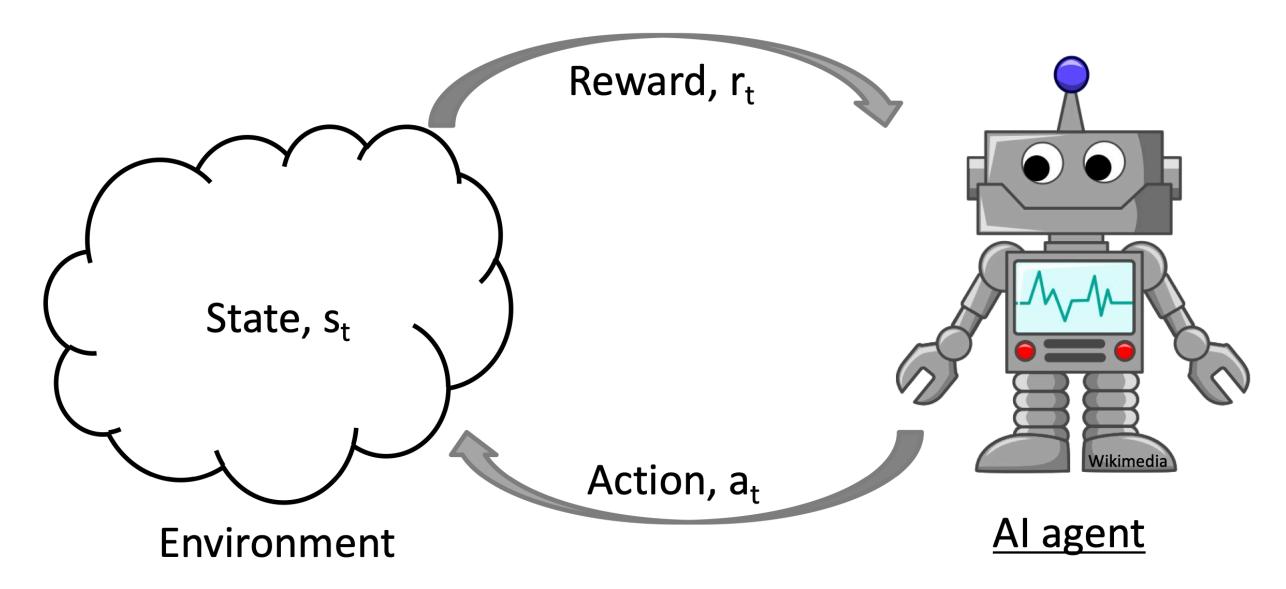
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- Unsupervised learning $\mathcal{D} = \{x^{(i)}\}_{i=1}^{N}$
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 - Dimensionality reduction
- Reinforcement learning $\mathcal{D} = \{s^{(t)}, a^{(t)}, r^{(t)}\}_{t=1}^{T}$ Since r

In many cases, we cannot precisely define what the correct output is (think of we want to train a robot to walk)

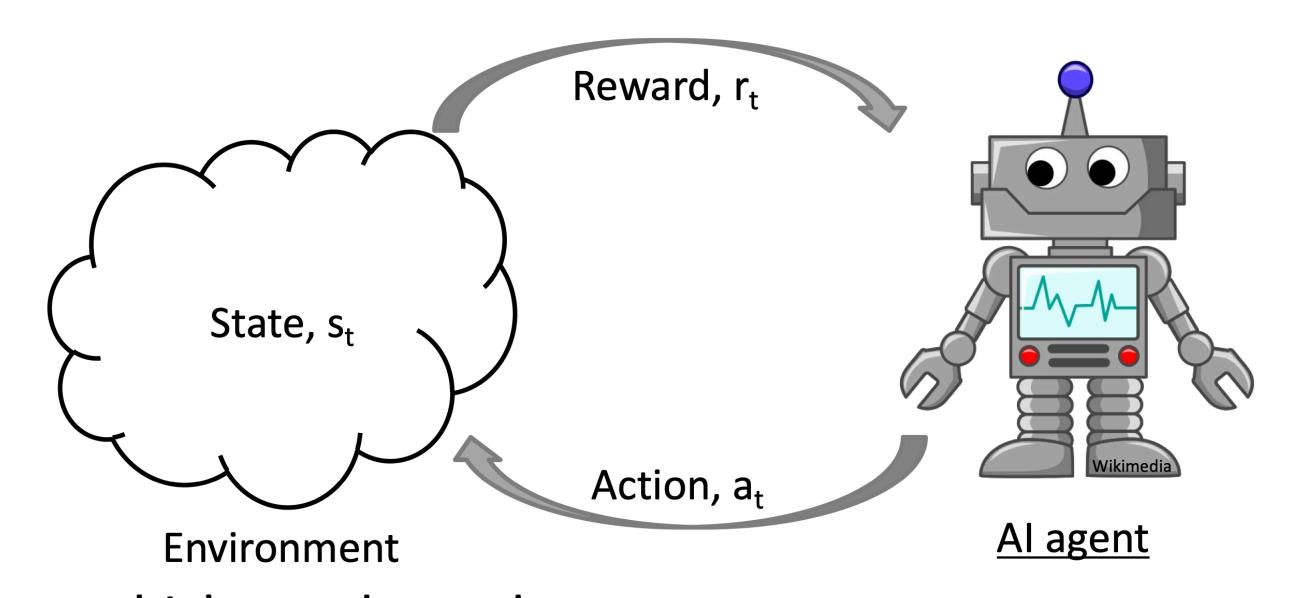


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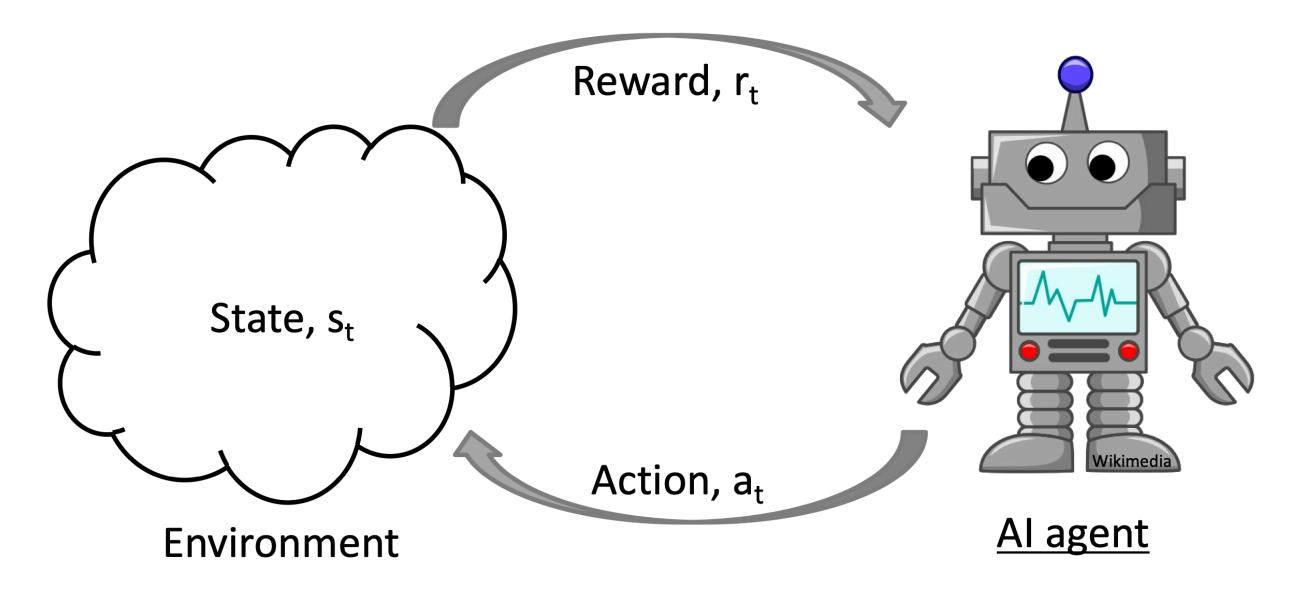
Agent chooses actions which can depend on past

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Agent chooses **actions** which can depend on past Environment can change **state** with each action

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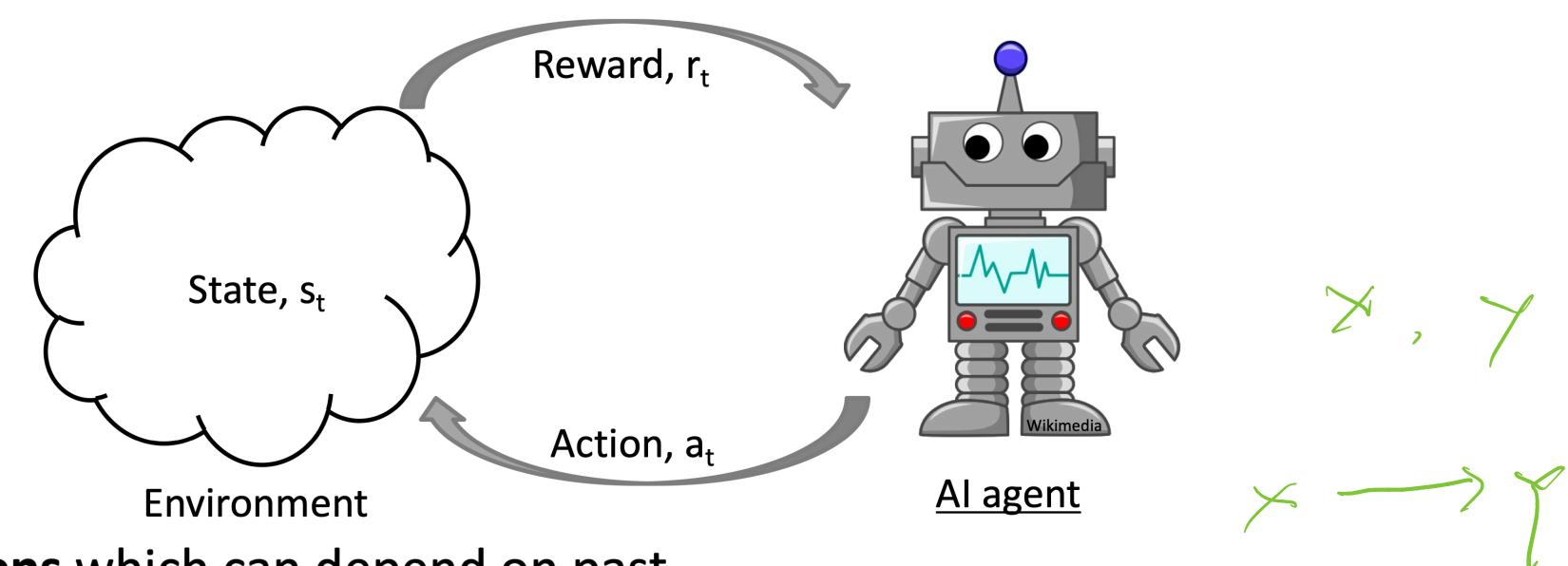


Agent chooses actions which can depend on past

Environment can change state with each action

Reward (Output) depends on (Inputs) action and state of environment

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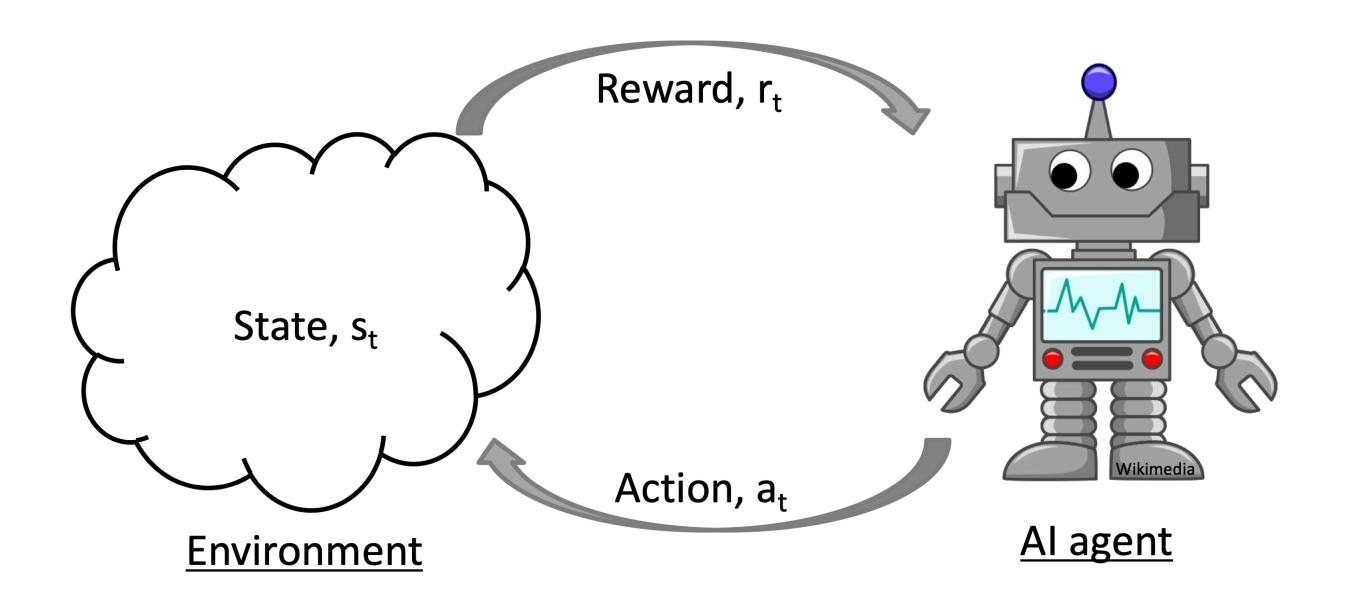


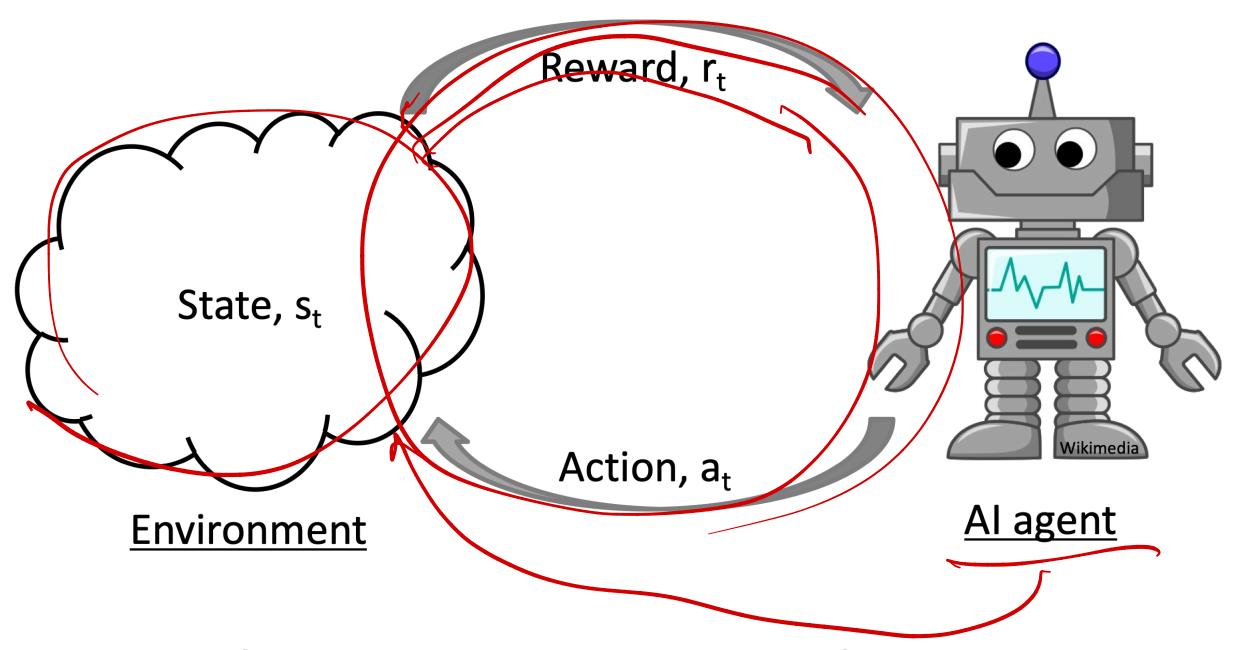
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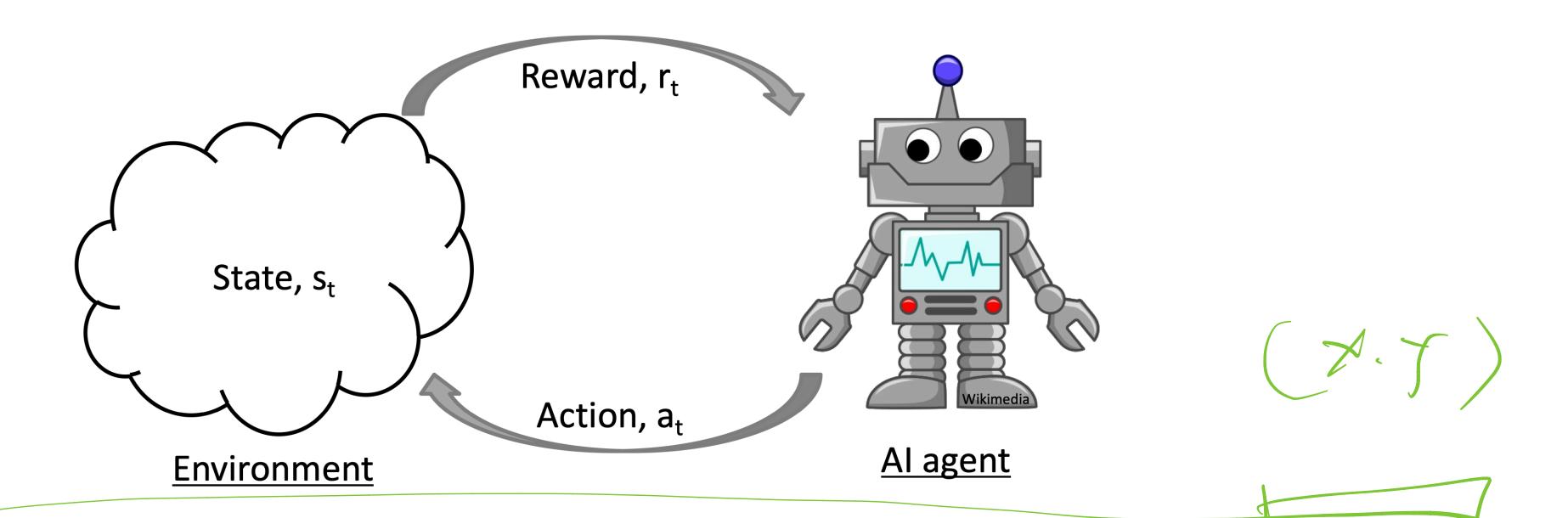
Reward (Output) depends on (Inputs) action and state of environment

Goal: maximize the total reward

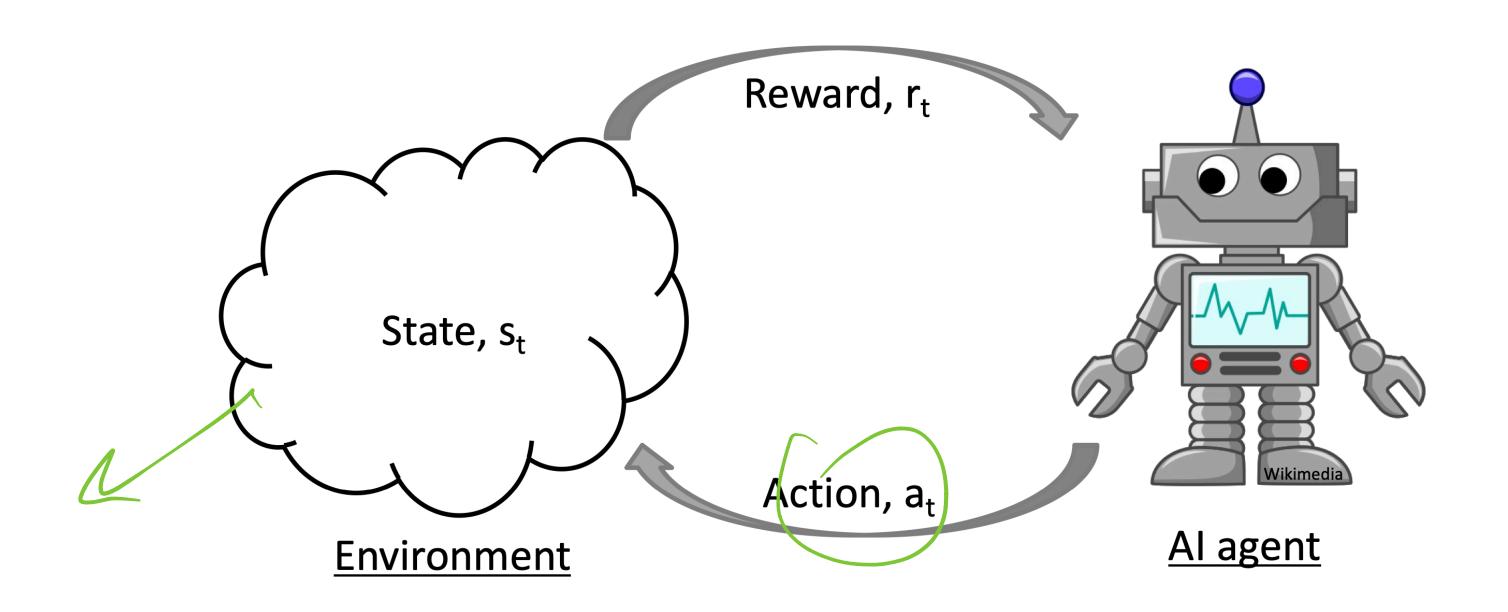




Maximize reward (rather than learn reward)



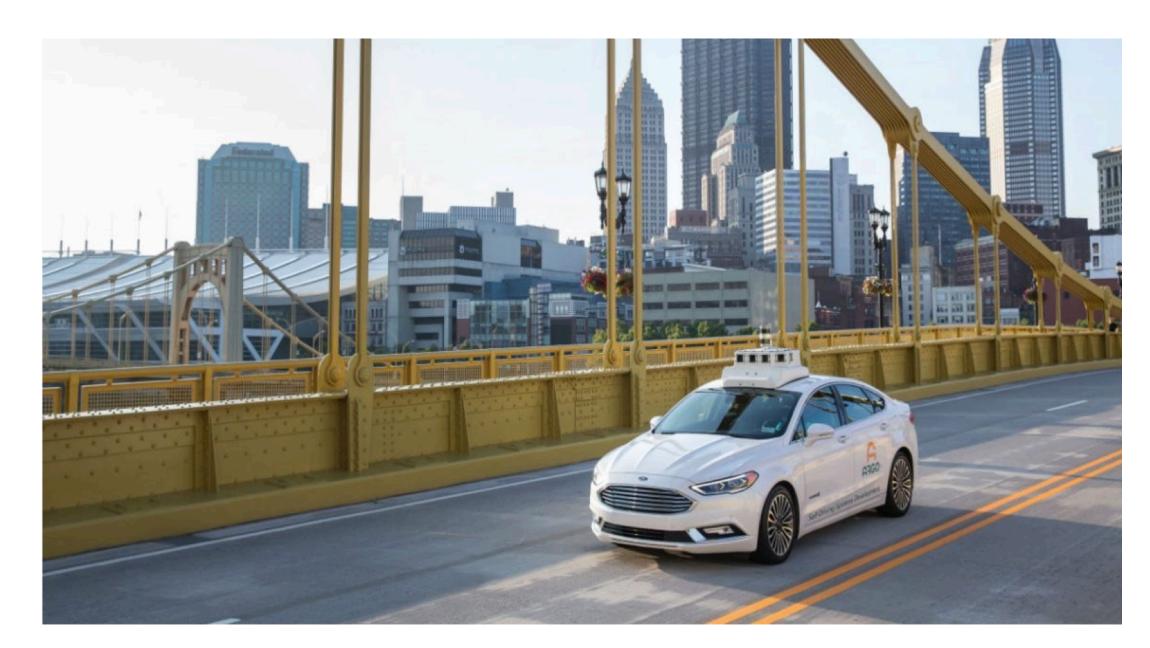
Maximize reward (rather than learn reward) Supervised training is like imitation



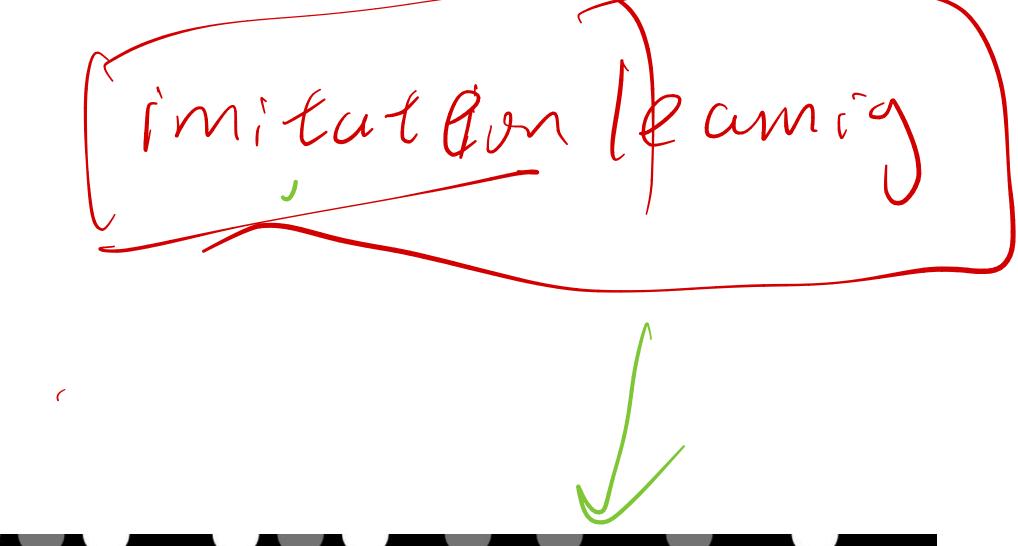
- O Maximize reward (rather than learn reward) Supervised training is like imitation
- Inputs are not iid state & action depends on past

RL Examples

 Cx_{i}

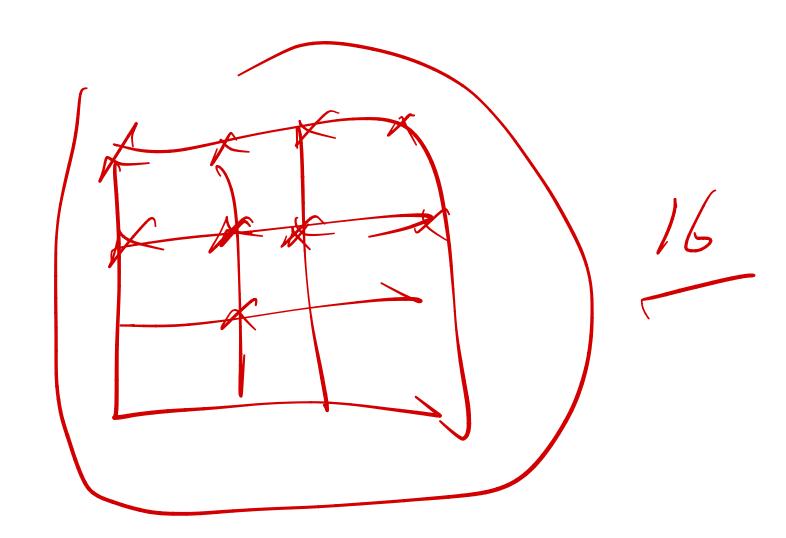


Reword





- State space, S
- Action space, A



- State space, S
- Action space, A
- Reward function
 - Stochastic, $p(r \mid s, a)$
 - Deterministic, $R: \mathcal{S} \times \mathcal{A} \to \mathbb{R}$

RES, A) -> Value

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- Transition function
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- Reward and transition functions can be known or unknown

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In this lecture, we assume they are known

Reward and transition functions can be known or unknown



Specifies an action to take in every state

- Policy, $\pi:\mathcal{S}\to\mathcal{A}$
 - Specifies an action to take in every state

- Value function, V^{π} : $\mathcal{S} \to \mathbb{R}$
 - Measures the expected total reward of starting in some state s and executing policy π , i.e., in every state, taking the action that π returns

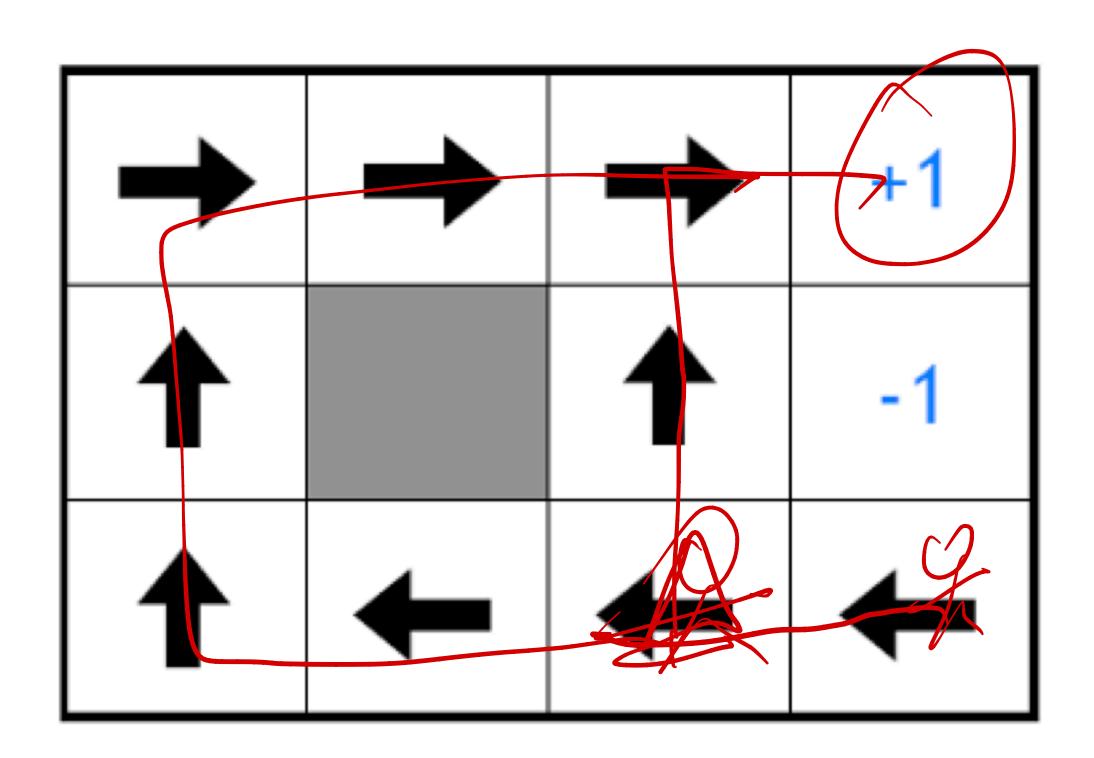
 $\mathcal{S}=$ all empty squares in the grid

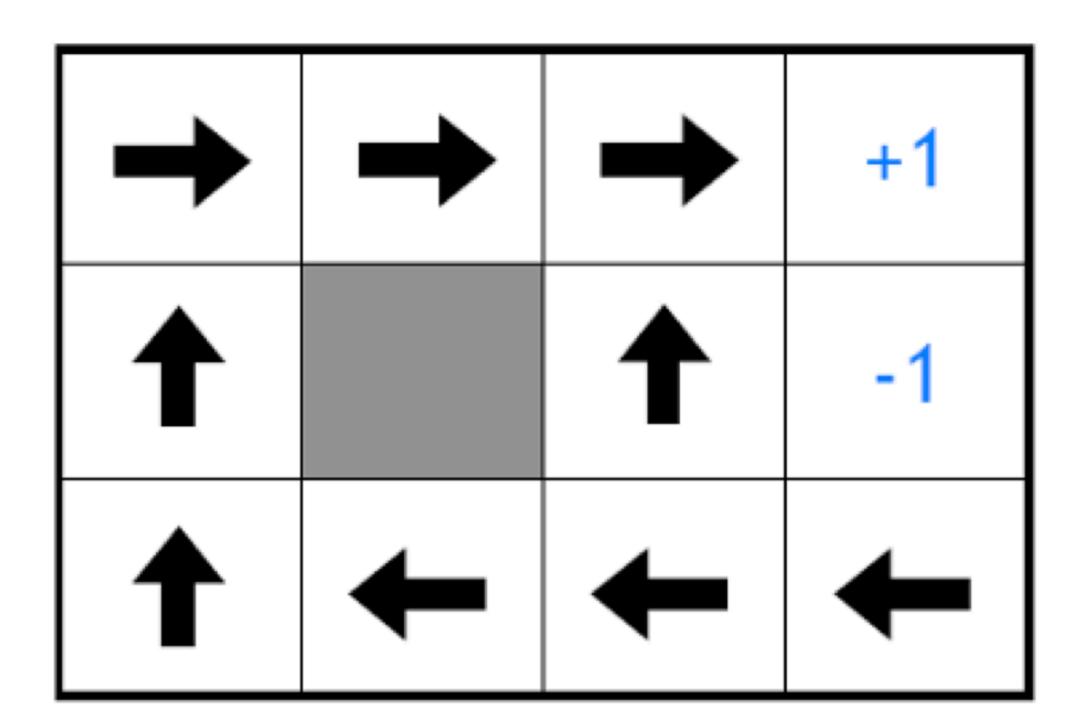
 $A = \{up, down, left, right\}$

Deterministic transitions

Rewards of +1 and -1 for entering the labelled squares

Terminate after receiving either reward



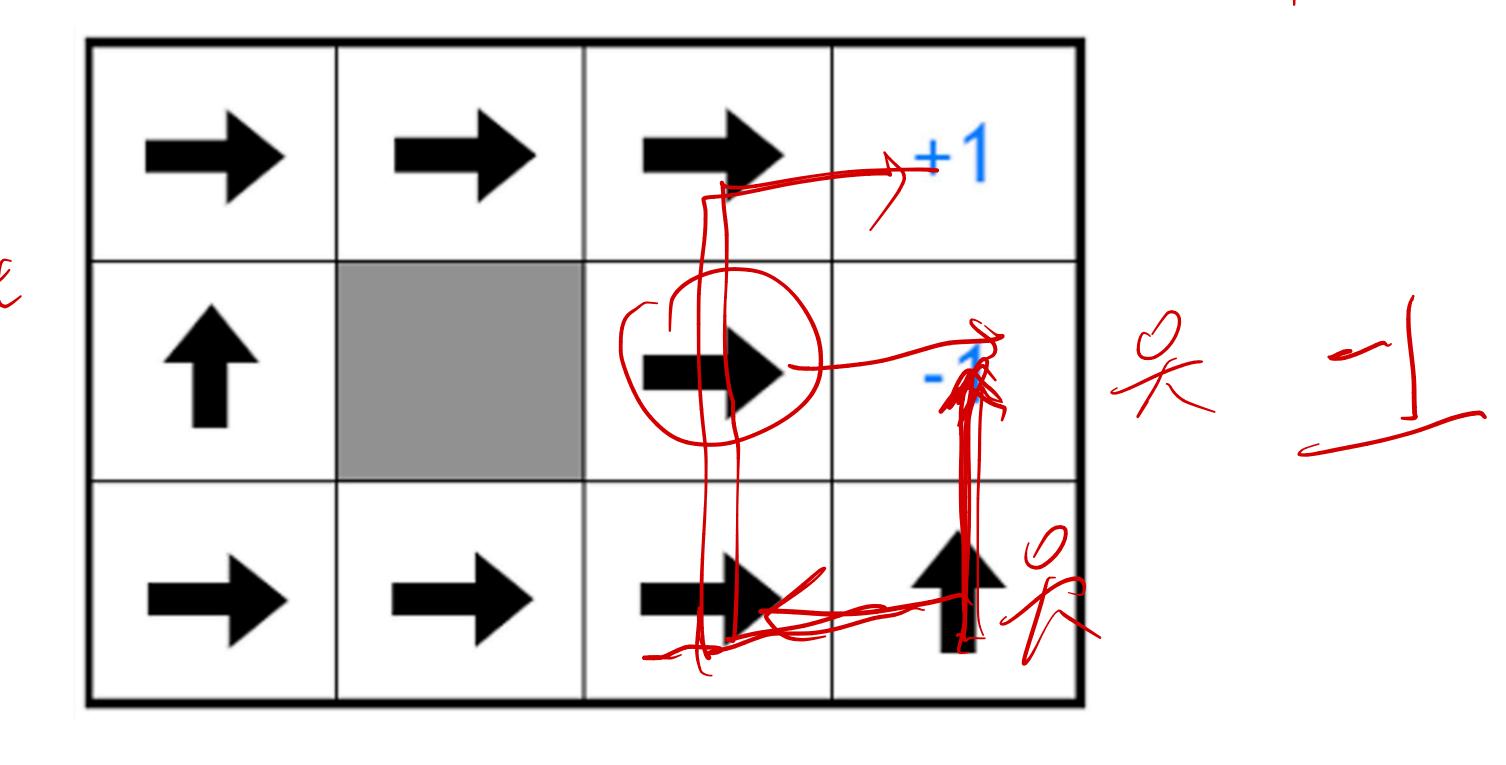


Is this policy optimal?

Optimal policy given a reward of -2 per step

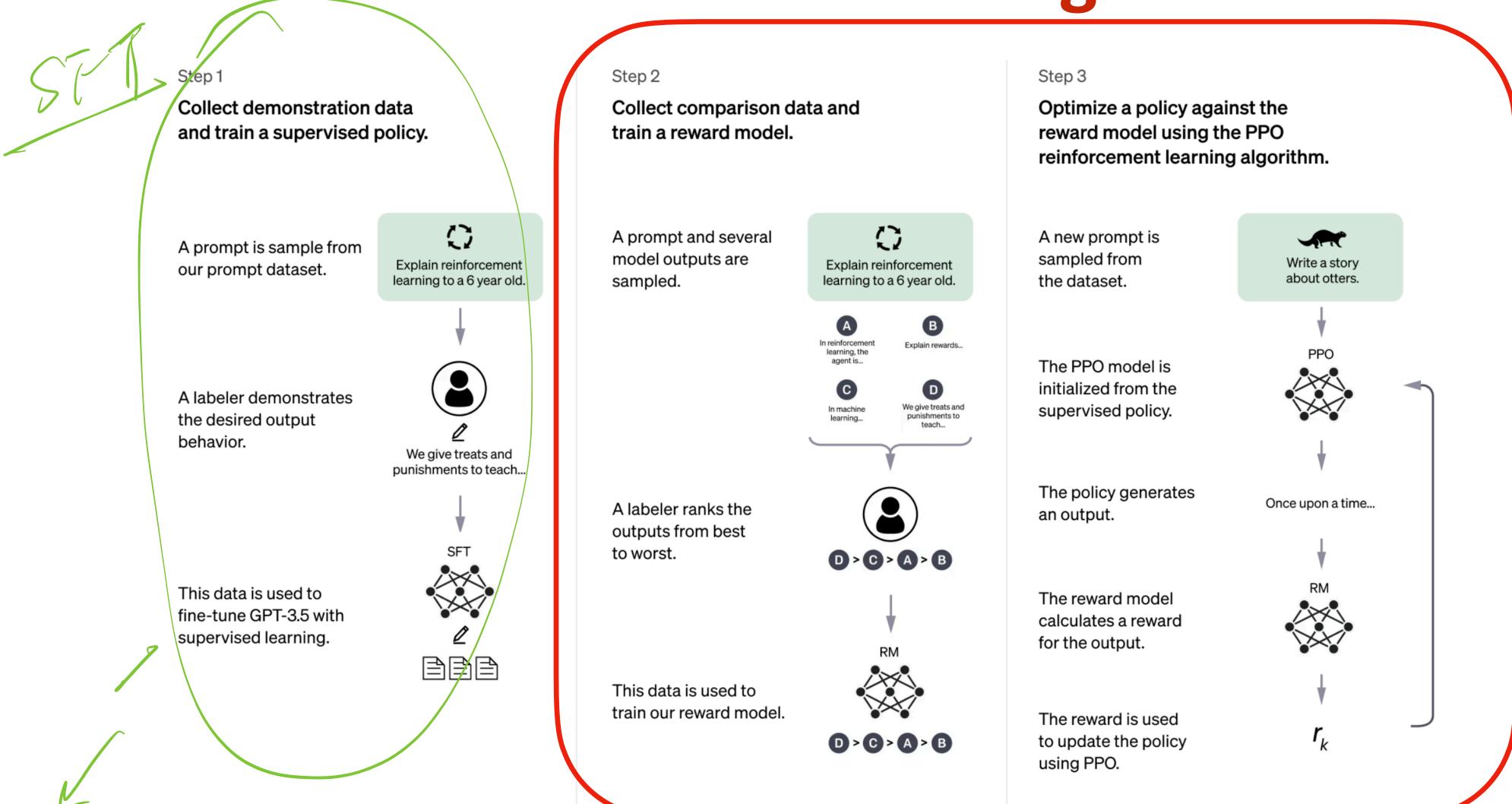
Optimal policy given a reward of -2 per step

-8+1:-7



-2 -1

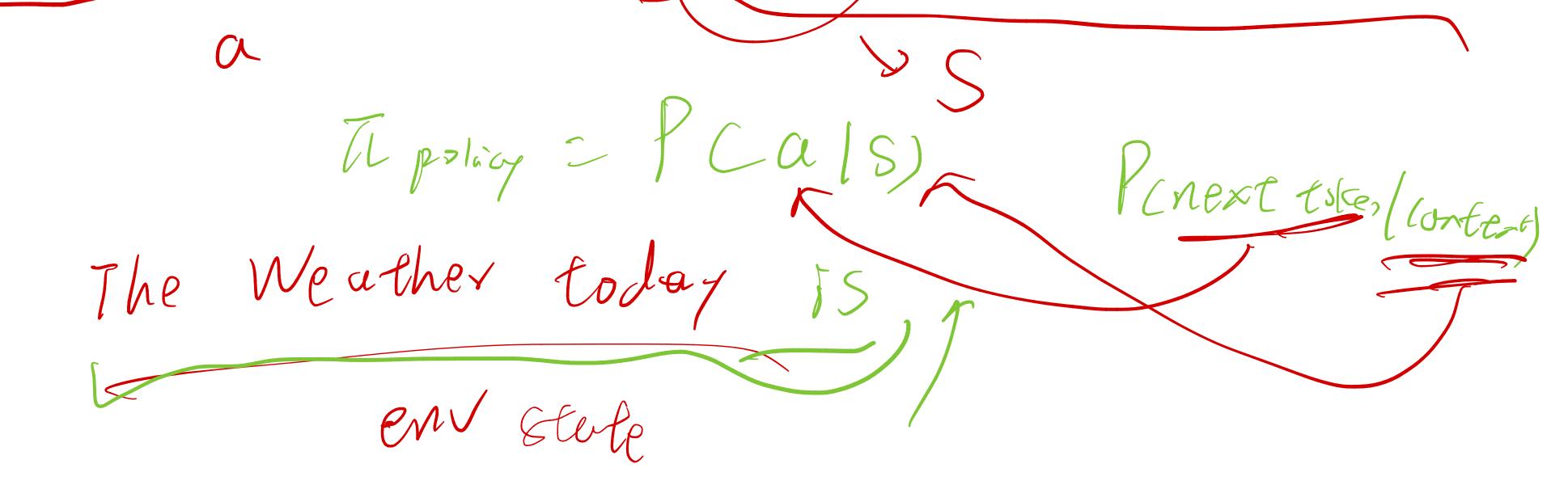
Reinforcement Learning in LLMs



Chal GP 7



- 1. There is no external environment
- 2. Each predicted token is an action, and the context is the environment states



From Imitation to Optimization

From Imitation to Optimization

Imitation (SFT)

Fit $\hat{p}(y|x) \approx p^*(y|x)$ for some reference distribution $p^*(y|x)$

- Pure generative modeling perspective
- Requires samples from reference policy

From Imitation to Optimization

Imitation (SFT)

Fit $\hat{p}(y|x) \approx p^*(y|x)$ for some reference distribution $p^*(y|x)$

- Pure generative modeling perspective
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Optimization (RLHF)

RLHT RL from human feedbook

Find $\widehat{p}(y|x)$ such that $\max_{p} E_{p}[R(y,x)]$ for a reward R(y,x)

- Maximize some reward function that we can measure
 - LMs are policies, not a model of some distribution

Pocx) Language Mode/

Objective:

$$\theta = \arg\max_{\theta} \mathbb{E}_{x \sim p_{\theta}(x)} R(x)$$

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SPI: 0: avy max []

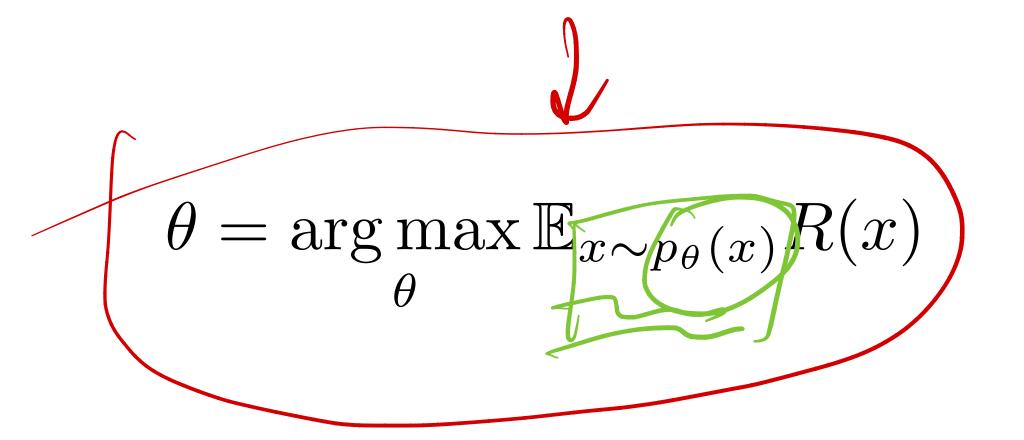
X is the language sequence, R is the reward function, scores the generated response (we can consider R as a

human or a model)

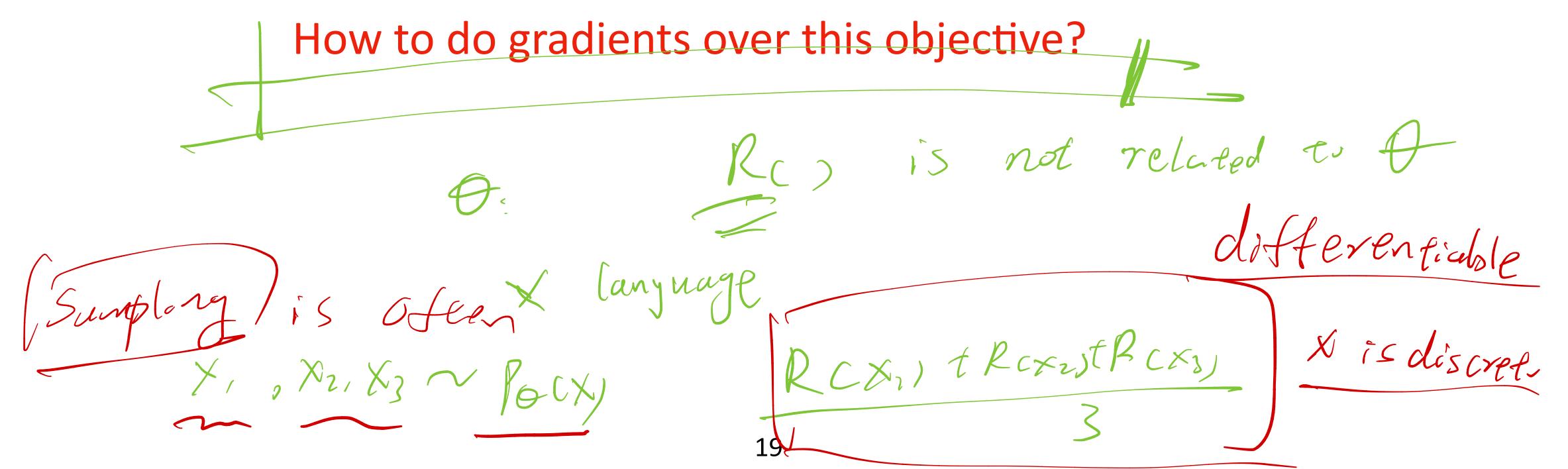
2 is human

mitate

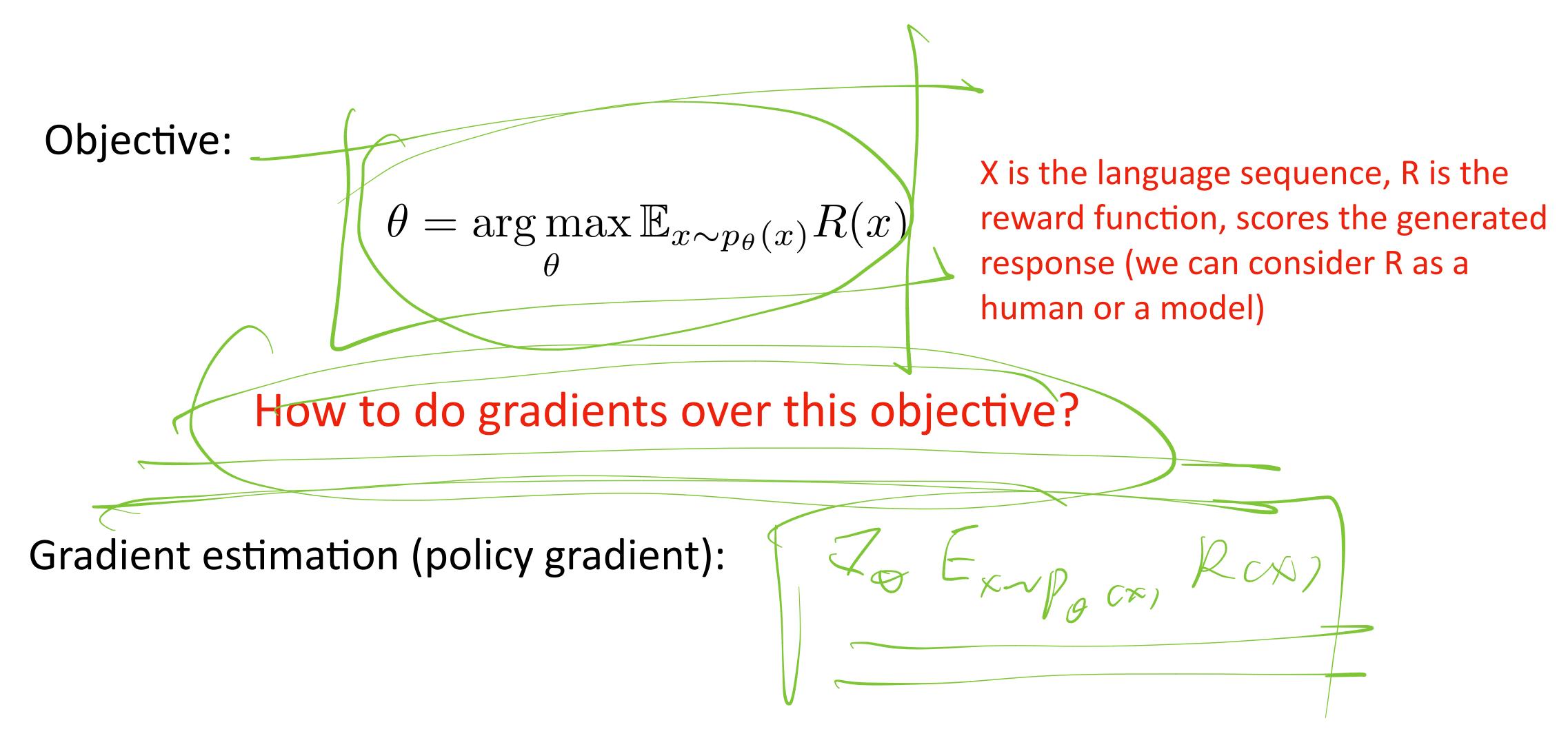




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Vandom Sumpling: Nc0,1) NCT, 1) X -> XA -> h -> hB-> h h=xA hi=hB 6chB)/



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How to do gradients over this objective?

Gradient estimation (policy gradient):

$$\hat{g} = \mathbb{E}_{x \sim p_{\theta}(x)} R(x) \nabla_{\theta} \log p_{\theta}(x)$$

19

 \hat{g} is the gradient (not objective)

Policy Gradient:
Objective: Ex-Poix, RCX)

To Exaposis, Rox = To I Pock Rox)

To Pocx): Pocx). To lay Pocx) = (Z) To Pocx) R cx)

20 Pocx, 2 (2 Pocx) Tolog Pocx, Rcx,

= Exmerx) PCK, ZeloyPock,

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How to implement?

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$$\text{How to implement?}) \neq \text{objective} = \hat{g}$$

$$\text{Objective} = \sum_{i=1}^{n} \frac{1}{R(x^{(i)})} \log p_{\theta}(x^{(i)})$$

$$x^{(1)}, ..., x^{(n)} \sim p_{\theta}(x)$$

$$\text{in o gradient}$$

$$\hat{g} = \mathbb{E}_{x \sim p_{\theta}(x)} R(x) \nabla_{\theta} \log p_{\theta}(x)$$
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$$\sum_{i=1}^{n} \frac{1}{n} R(x^{(i)}) \log p_{\theta}(x^{(i)})$$

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This objective looks kinda like weighted log likelihood maximization?

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- 1. Have a weight of R(x)
- 2. The data x is sampled from the model itself, not from a static dataset

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This is the simplest form of RL, many other RL algorithms (PPO, GRPO) are more like variants of this simple equation with the same spirit

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$$X^{(1)}, \dots, X^{(n)} \sim p_{\theta}(x)$$
Have a weight of R(x)

The relative interest of forces the rest of the set forces to the set of the se

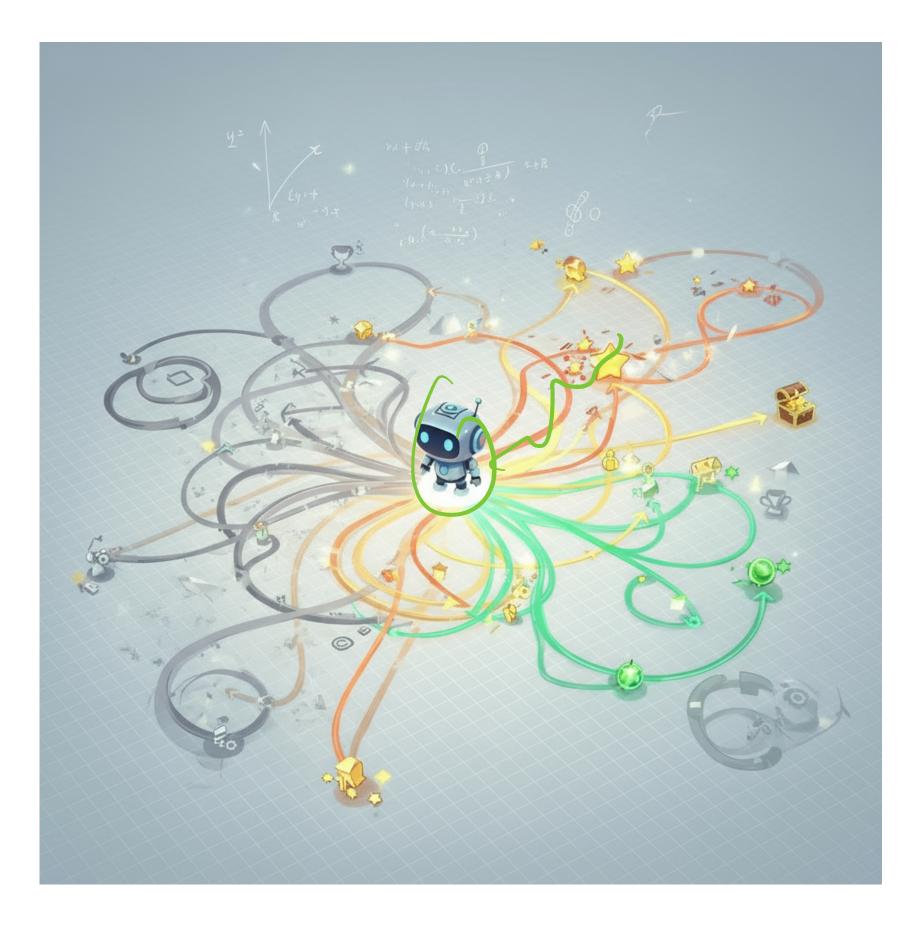
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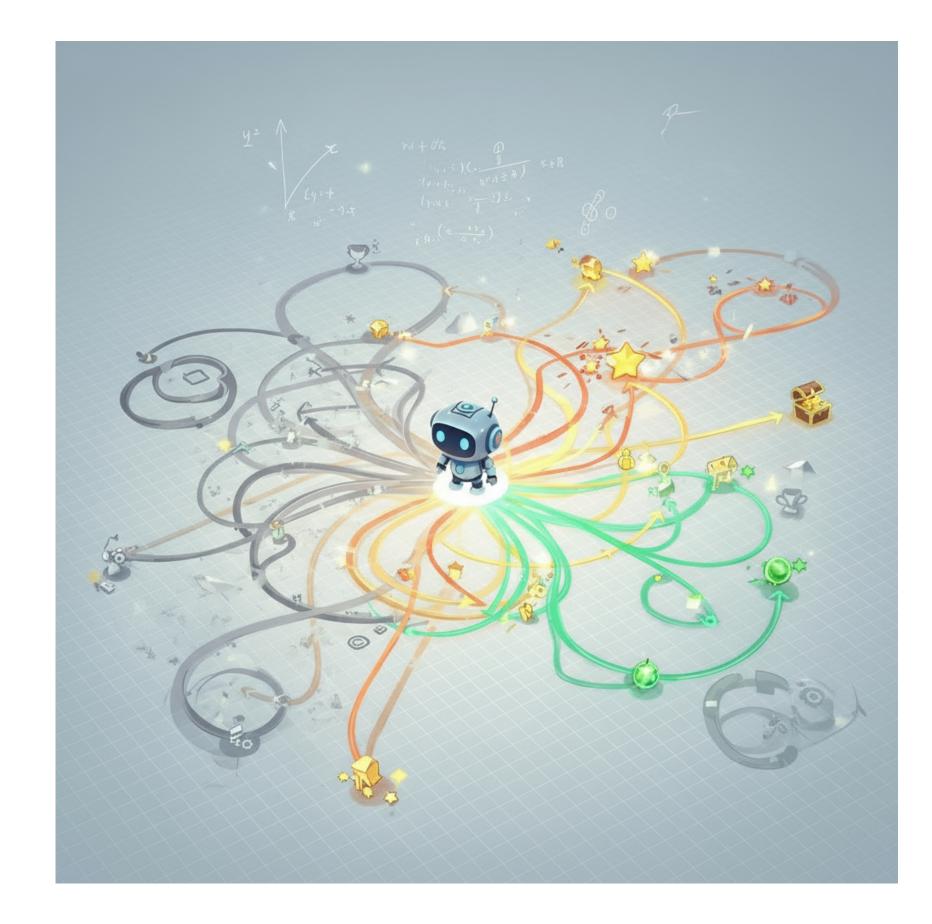
We can see why RL is called "self-improving", and it is trained by "synthetic data"

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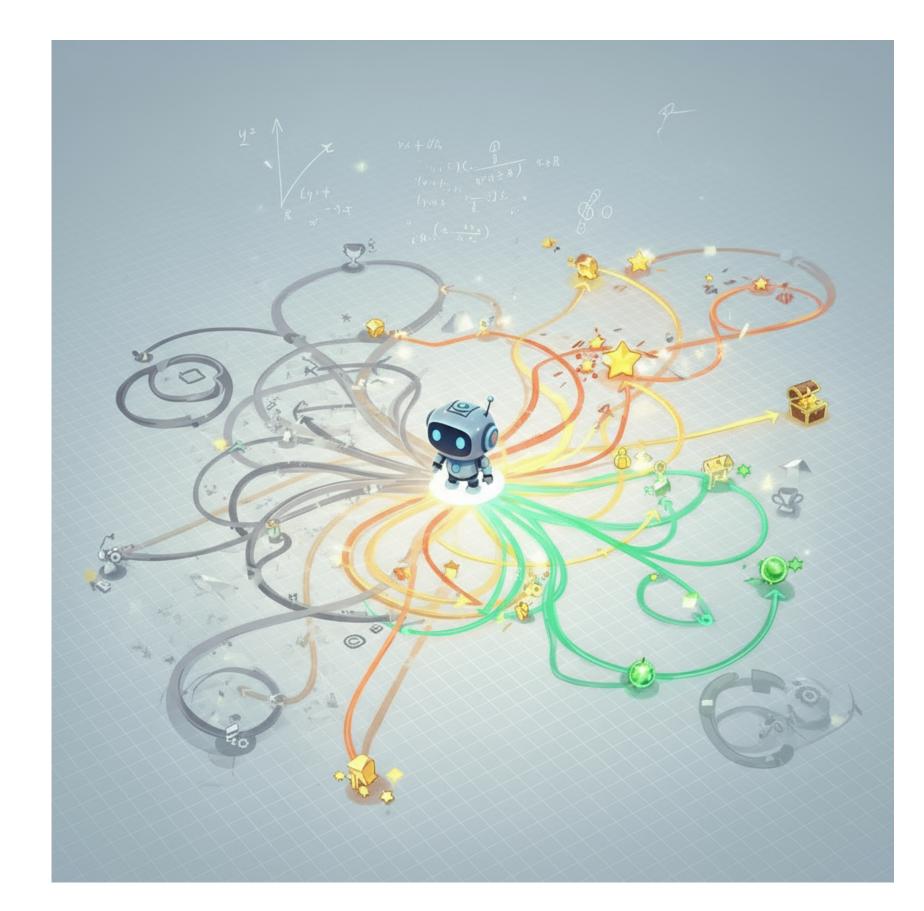
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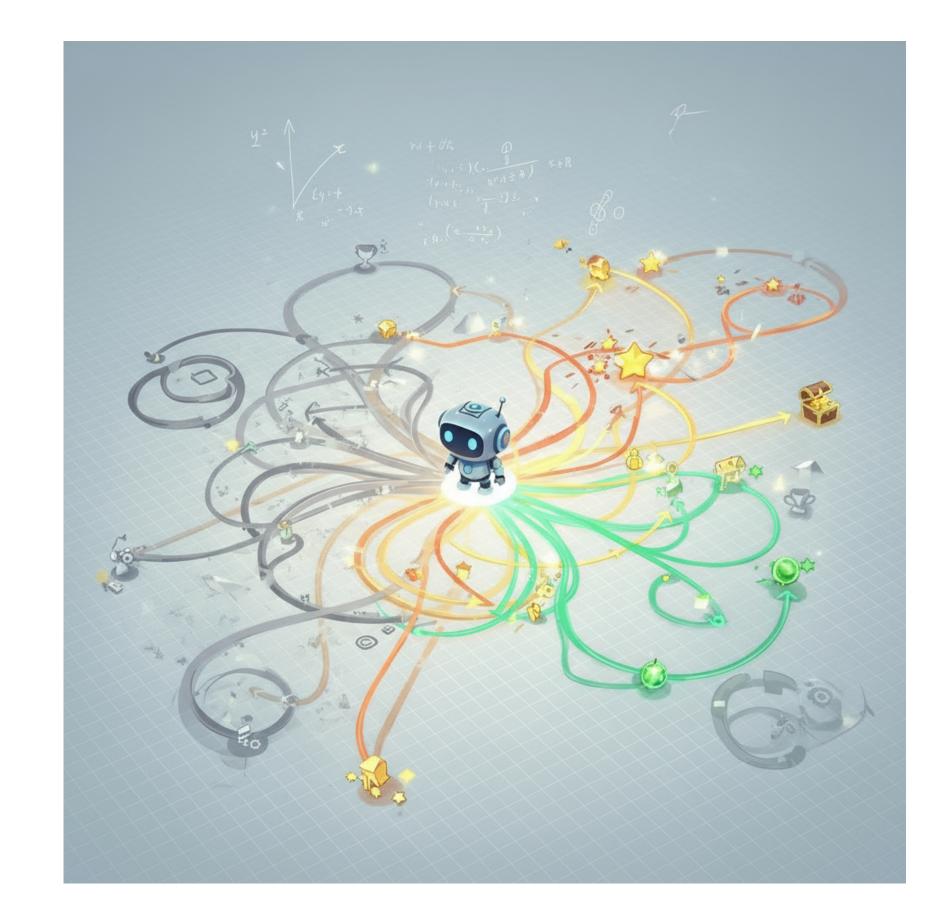
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- 1. Sample data from the model (or we call policy) itself (exploration)
- 2. A reward function judges whether the explored data is good or bad



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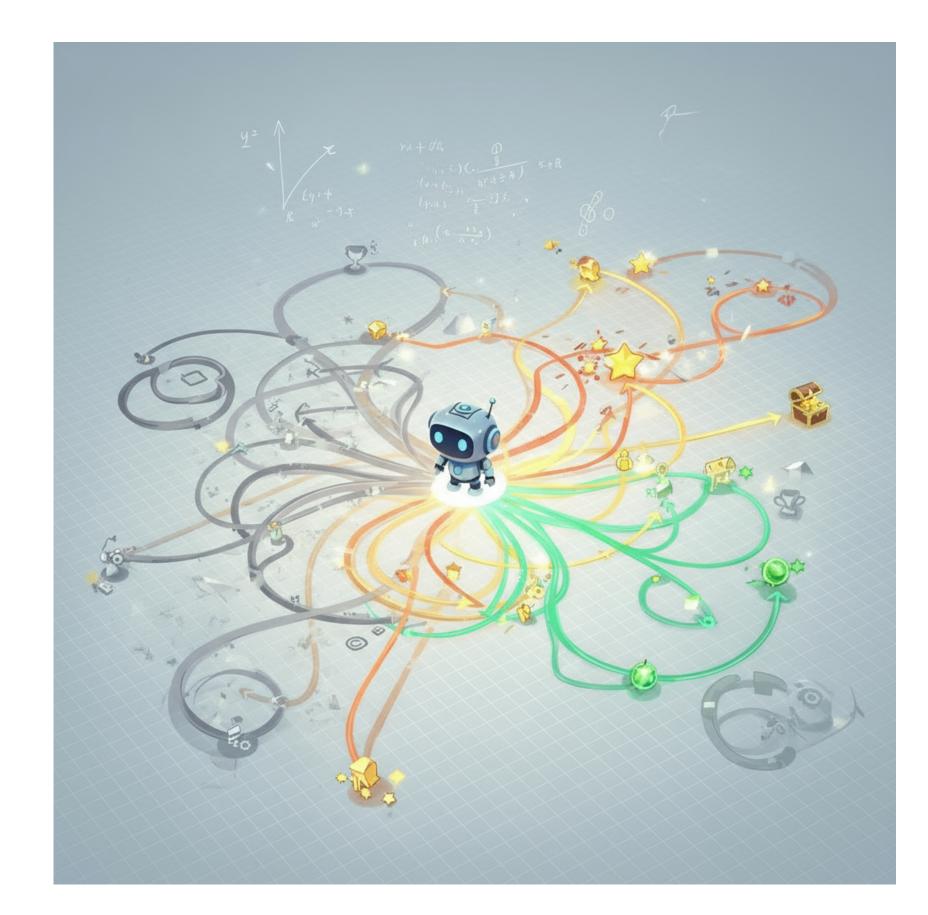
Reinforcement learning is a mixed art of both training and inference during training time



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Why?



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 Inference

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 Inference

Each training step, the algorithm needs to run inference again $\rho_{\theta_{eff}}(x)$

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 Inference

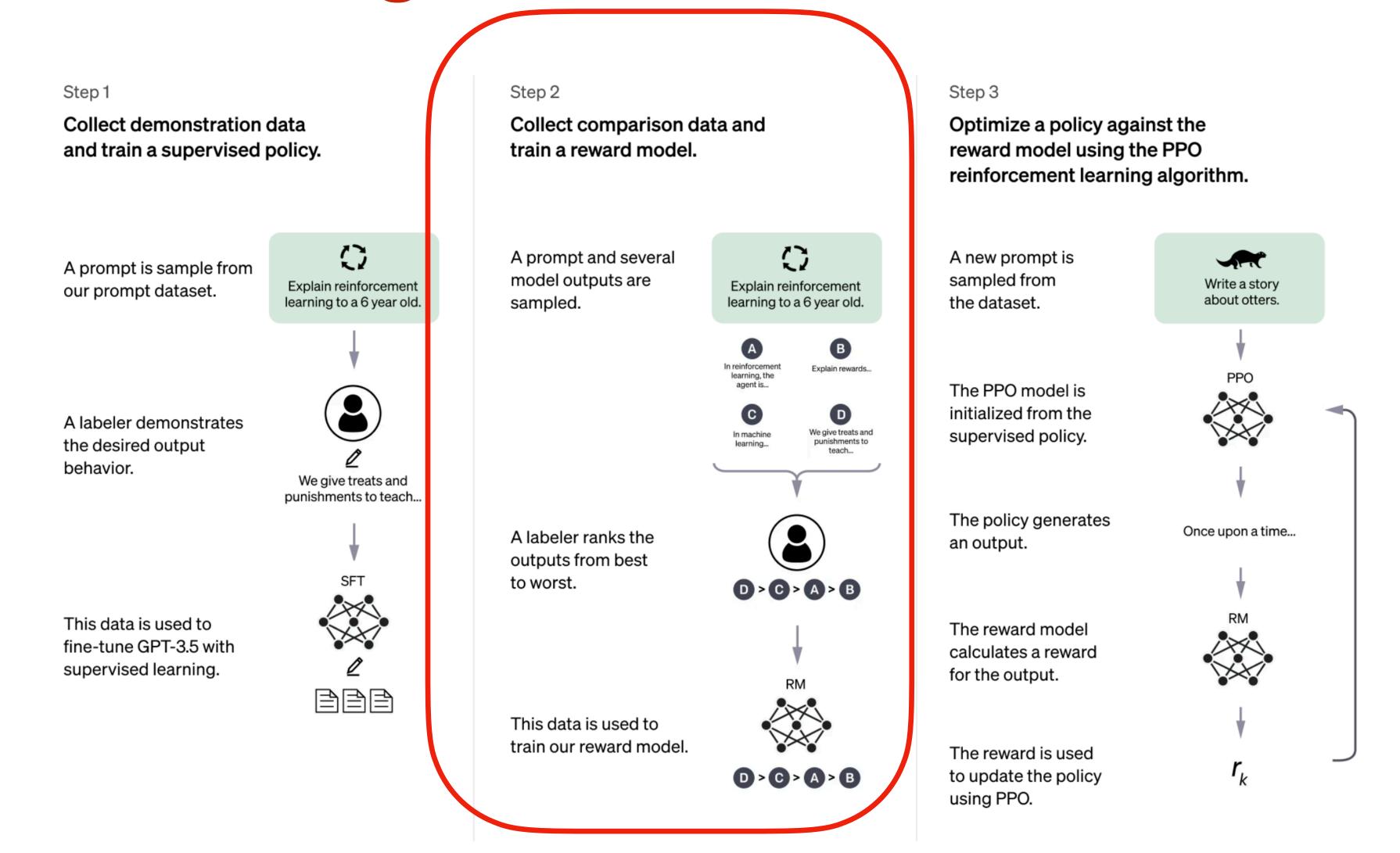
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REINFORCE / Policy Gradient for Language Models

Objective =
$$\sum_{i=1}^{n} \frac{1}{n} R(\{x_1^{(i)}, x_2^{(i)}, x_t^{(i)}\} \sum_{j=1}^{t} \log p_{\theta}(x_j^{(i)} | x_{< j}^{(i)})$$

Training a Reward Model in RLHF



Training a Reward Model

Suppose we have K responses and have them ranked by humans, then for all possible pairs of responses, y_w is the preferred one, y_l is the less preferred one, the objective of reward model $r_{\theta}(x, y)$ is:

Objective =
$$-\frac{1}{\binom{K}{2}} E_{(x,y_w,y_l)\sim D} \left[\log \left(\sigma \left(r_{\theta}\left(x,y_w\right) - r_{\theta}\left(x,y_l\right)\right)\right)\right]$$

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In practice, we can just sample a subset of pairs, rather than enumerating all pairs

	Base model -	Supervised learning (one time)	→ Pairwise feedback -	► RL	→ Evaluation
Compute cost	\$300k	\$100	\$100	\$100	\$0
Annotation cost	\$0	\$25k	\$4k	\$0	\$50

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- 1. SFT data can be expensive
- 2. There may be tasks that are much easier for humans to verify than solve, particularly when AI today is outperforming humans

Reinforcement Learning from Human Feedback Data

Basic Setup

Your annotation progress for this HIT: 0/6 completed. Sample 1 of 6. Lets you skip forward (all samples still need to be marked): → Next sample. Returns you back to the next un-marked sample to mark: M Go to next un-marked sample. Annotation Guidelines Instruction Tell me about self driving cars Input Al Response 1 Self driving cars are vehicles that are capable of detecting their surroundings and navigating without any input from a human driver. They use a variety of sensors, such as cameras, radar, and lidar, to detect objects in their environment and use advanced algorithms to make decisions about how to maneuver around them. Self driving cars are becoming increasingly popular and are being used in a variety of applications, such as ride-hailing services and autonomous delivery services. Al Response 2 Self driving cars are cars that are equipped with sensors, cameras, and software, which allows them to navigate without the need for a driver. Self driving cars are able to monitor their environment and navigate without the assistance of a driver. Self driving cars use a combination of technologies such as GPS, radar, lidar, and computer vision to detect their environment and avoid obstacles. They are also able to update their maps and adjust their routes to account for variables such as traffic and construction. The main benefit of self driving cars is their ability to reduce crashes and save lives. Autonomous vehicles have the potential to save up to 30,000 lives annually. They are also more energy efficient than traditional vehicles, as they do not require a driver who has to maintain a minimum speed. Autonomous vehicles also create new opportunities for increased mobility, allowing those who are unable to drive to get around. Rating Response 1 is better. OResponse 1 is only slightly better. (only pick this if it's truly close) Response 2 is only slightly better. (only pick this if it's truly close) Response 2 is better.

Hi test22. Not your MTurk id? Logout

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Easier than annotating the responses directly

Thank You!