Lecture 17 HMM

$P(h)$ 7 latent variables

k

$\sum_{a \cdots g} P(a, b, c, \ldots h)$ $O(k^7)$

$P(x_1, x_2 \cdots x_4) = P(x_1) P(x_2|x_1)$

$P(x_3|x_2) P(x_4|x_3)$

x_3 and x_1 conditionally independent given x_2

$O(7)|c^2|$

$a \cdots b \cdots g$

$P(a, b, c, d, e \cdots g, h)$

$= \sum_a \sum_b \sum_c \cdots \sum_g P(a) P(b|a) P(c|b) P(d|c) \cdots$

$= \sum_b \sum_c \cdots \sum_g \left[\sum_a P(a) P(b|a) \right] P(c|b) P(d|c) \cdots$

$O(|c^2|)$ ← $f(b)$

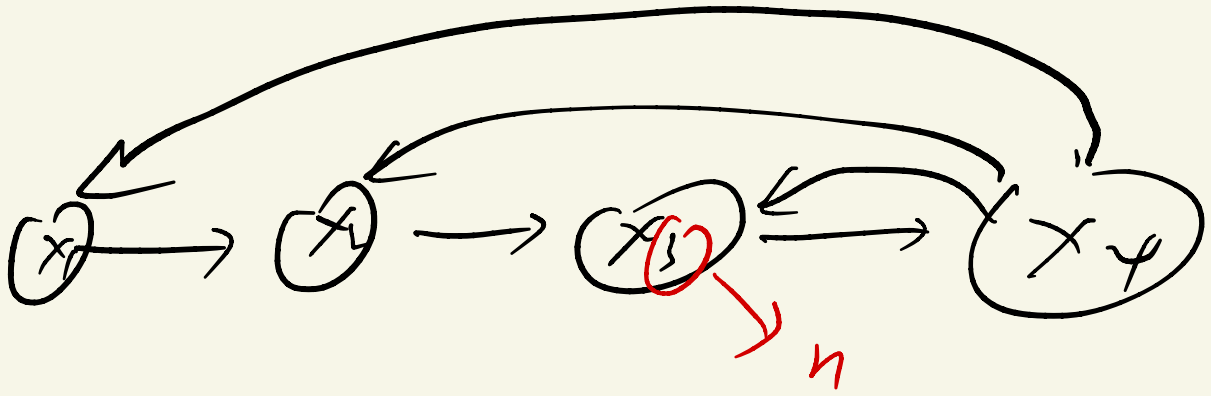
$$P(h) = \sum_a \sum_b \cdots \sum_g P(a) P(b|a) \cdots$$

$$a, b \cdots g = \arg\max_a \arg\max_b \cdots \arg\max_g P(a) P(b|a) \cdots$$

$$\max_a \cdots \max_g P(a) P(b|a) P(c|b)$$

$$\max_b \cdots \max_g \left[\max_a P(a) P(b|a) \right] P(c|b)$$

$f(b)$



$$P(x_1 \cdots x_4) = P(x_1)\, P(x_2 \mid x_1)$$

$$P(x_3 \mid x_1, x_2)$$

$$P(x_4 \mid x_1\ x_2\ x_3)$$

$$P(x_3 = i \mid x_2 = j) \quad \left[\quad \right]^{k}$$

$$= P(x_4 = i \mid x_3 = j) = P(x_{t} = i \mid x_{t-1} = j)$$

$$P(O_1 \cdots O_T) = \sum_S P(O, S)$$

$$P(x)$$

$$P(x) = \sum_z P(x, z)$$

$K \times K$

$K \times M$

$$\prod_{t=1}^{T} P(S_t | S_{t-1}) \cdot P(O_t | S_t)$$

M.G

$P(S_1)$ K O has M

$$P(x, z)$$

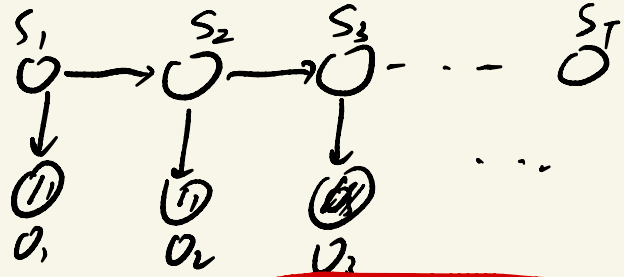
O_L and O_T independent

given S_1? No

given S_2? Yes

given S_3? Yes

$$P(O_{1\cdots T}) = \sum_{S_1}\sum_{S_2}\cdots\sum_{S_T} P(O_{1\cdots T}, S_1, S_2 \cdots S_T)$$



$$= \sum_{S_1}\cdots\sum_{S_T} P(S_1)\,P(S_{-1}|S_1)\,P(O_1|S_1)\,P(S_2|S_2)$$

$$P(O_3|S_3)\,P(S_4|S_3)\cdots$$

$$\cdots \sum_{S_2}\sum_{S_3}\cdots\sum_{S_T}\left[\sum_{S_1} P(S_1)\,P(S_2|S_1)\,P(O_1|S_1)\right]\cdots$$

$$P(S_3|S_{-1})$$

S_T $f(S_{T-1})$ $f(S_2)$

$$\alpha_t^k = P(o_1 \cdots o_t, s_t = k)$$

$$P(o_1 \cdots o_T) = \sum_k P(o_1 \cdots o_T, s_T = k)$$

$$= \sum_k \alpha_T^k$$

$\alpha_1^k = P(o_1, s_1 = k)$

$$\alpha_t^k = P(o_1 \cdots o_t \cdots o_t, s_t = k)$$

$\alpha_T^k \quad \alpha_{T-1}^k \quad \alpha_{T-1}^{k'}$

$$= \sum_i P(o_1 \cdots o_{t-1}, o_t, s_{t-1} = i, s_t = k)$$

$$= \sum_i P(o_1 \cdots o_{t-1}, s_{t-1} = i) P(o_t, s_t = k \mid o_1 \cdots o_{t-1}, s_{t-1} = i)$$

$$= \sum_i P(o_1 \cdots o_{t-1}, s_{t-1} = i) P(s_t = k \mid s_{t-1} = i) P(o_t \mid s_t = k)$$

α_{t-1}^i

$$P(S_t = K \mid O_1 \cdots O_T)$$

$$= \underbrace{P(O_1 \cdots O_t, S_t = K)}_{\alpha_t^K} \underbrace{P(O_{t+1}, \cdots O_T \mid S_t = K)}_{\beta_t^K}$$

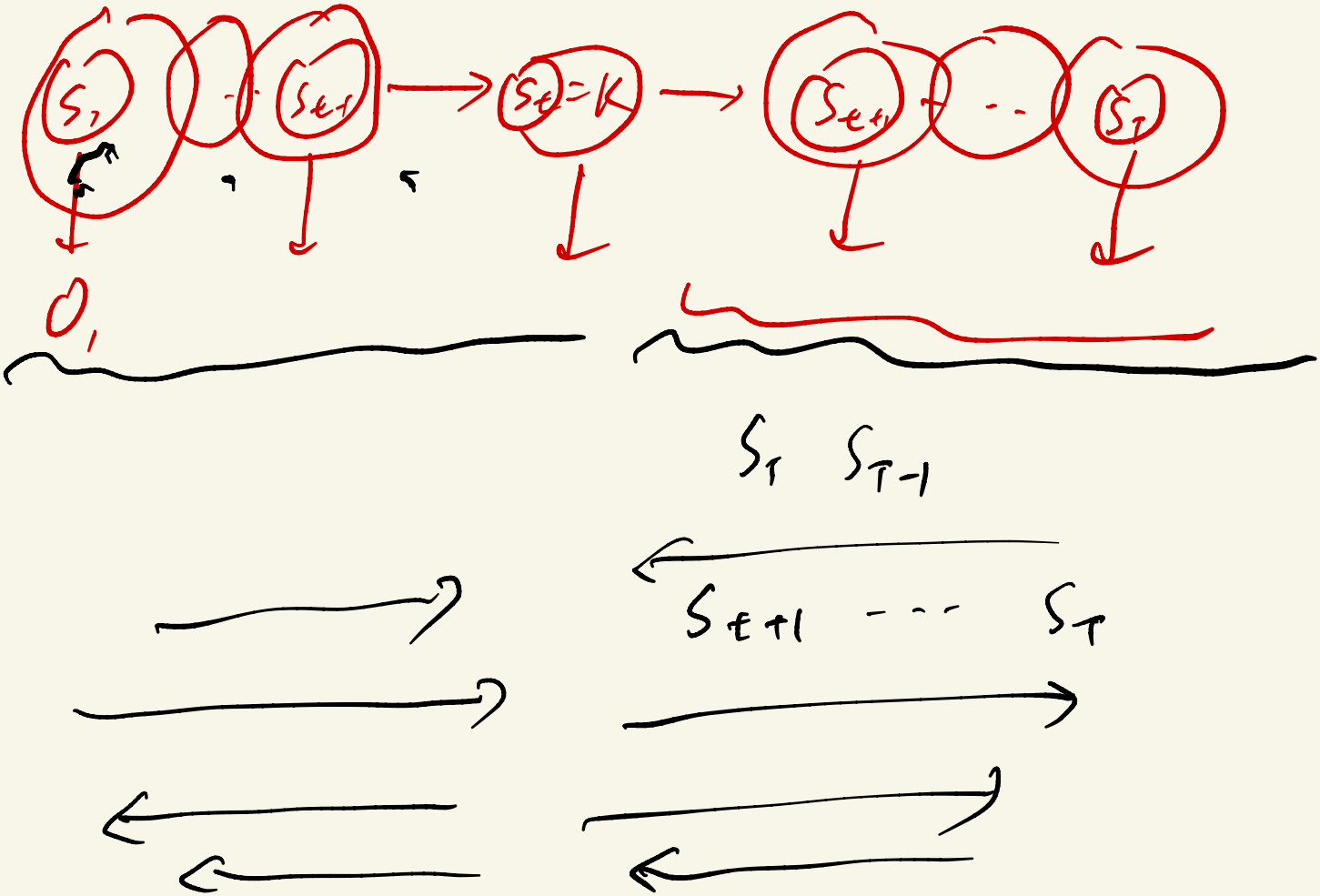
$$P(S_t = K, O_1 \cdots O_T) = \sum_{S_1} \sum_{S_2} \cdots \sum_{S_{t-1}} \sum_{S_{t+1}} \cdots \sum_{S_T}$$

$$P(S_1, S_2 \cdots S_t = K, S_{t+1} \cdots S_T, O_1 \cdots$$

$$P(S_t = K \mid O)$$

$$\propto P(S_t = K, O_1 \cdots O_T)$$

$$= \sum_{S_1} \sum_{S_2} \cdots P(S_1) P(S_2 \mid S_1) \cdots P(O_1 \mid S_1) \cdots O_T)$$


S_1
S_{t-1}
$S_t = K$
S_{t+1}
S_T
O_1
S_T S_{T-1}
S_{t+1} S_T

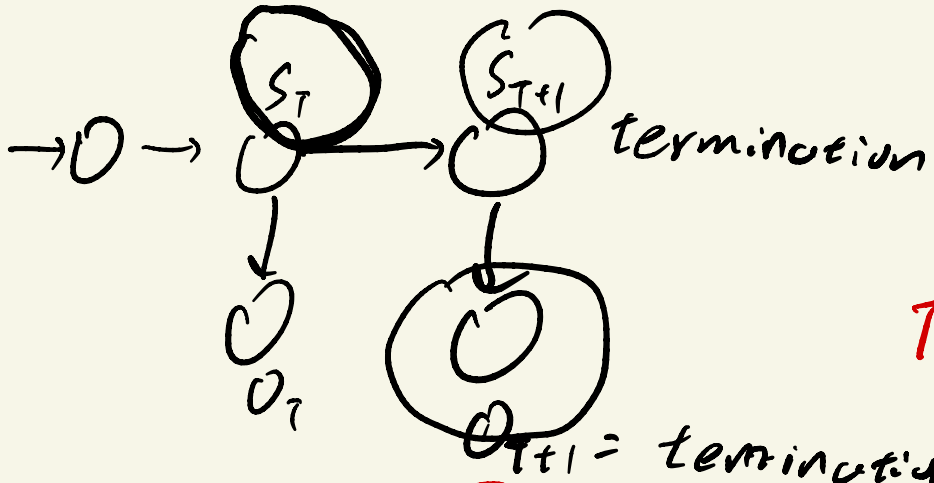
$$\beta_t^k = P(O_{t+1} \cdots O_T \mid S_t = k) \qquad \beta_{t+1}^k \qquad O_{T+1}$$

$$= \sum_{S_{t+1}=i} P(O_{t+1} \cdots O_T, S_{t+1}=i \mid S_t = k)$$

$$= \sum_{S_{t+1}=i} P(O_{t+2} \cdots O_T, S_{t+1}=i \mid S_t = k)\, P(O_{t+1} \mid S_{t+1}=i)$$

$$= \sum_{S_{t+1}=i} P(O_{t+2} \cdots O_T \mid S_{t+1}=i)\, P(S_{t+1}=i \mid S_t = k)\, P(O_{t+1} \mid S_{t+1}=i)$$

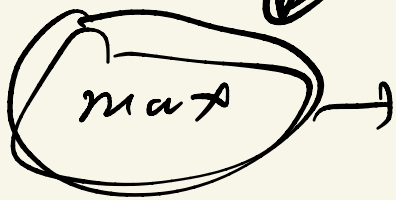
β_T^k $\qquad \beta_{t+1}^i$



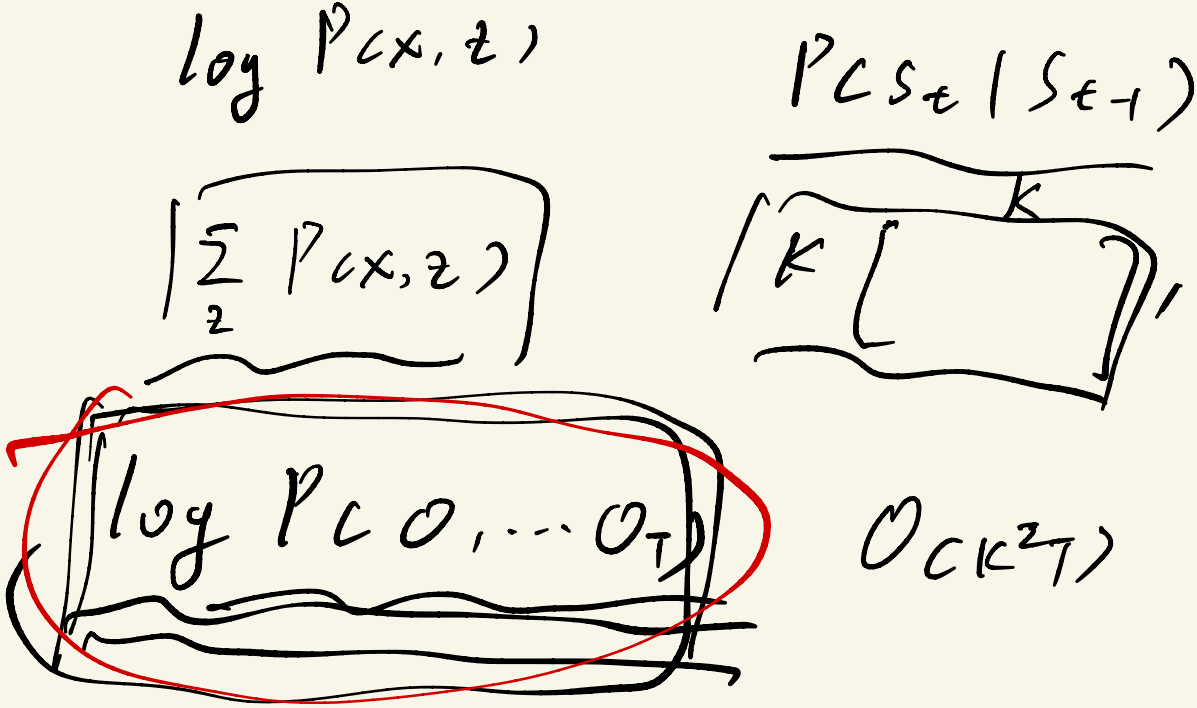
T

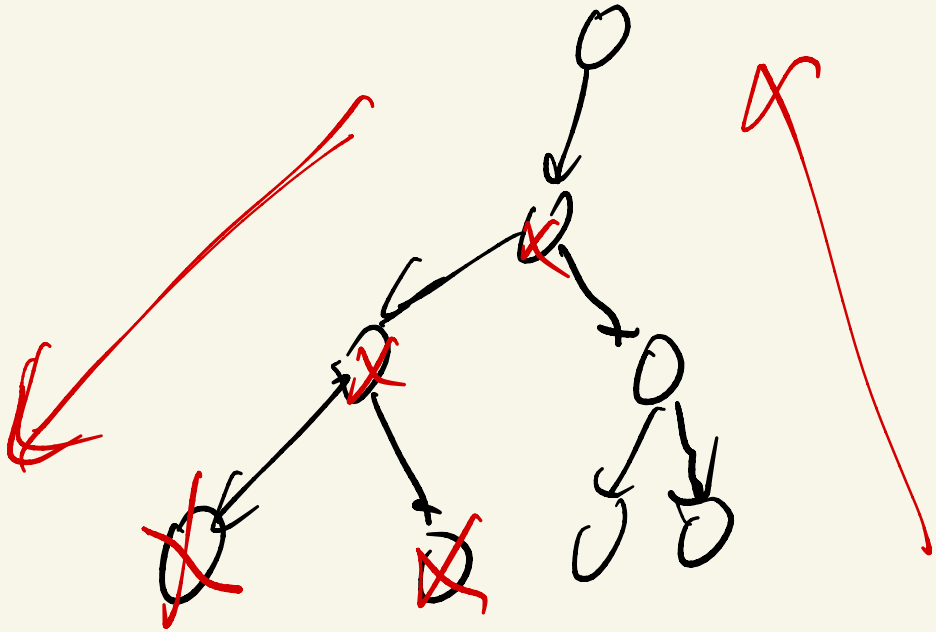
$$\beta_T^K = P(O_{T+1} \overset{\text{f(termin.)}}{=} \quad | \; S_{T=K})$$

$$P(O_1 \cdots O_T) = \sum_{S_1} \sum_{S_2} \cdots \sum_{S_T} P(S_1 \cdots S_T, O_1 \cdots O_T)$$



argmax


log P(x, z)
P(s_t | s_{t-1})
Σ_z P(x, z)
K
K
log P(o_1, ... o_T)
O(K^2 T)



$P < 0$