Lecture 21 VAE

$$z \sim p(z), \quad \underline{\mathcal{N}(0,1)}$$

$$x \sim p(x; f(z;\theta))$$

$$x \sim \mathcal{N}(\mu, \sigma^2)$$

$$\mu = f(z;\theta)$$

$$\sigma^2 = f_2(z;\theta)$$

$$P(x) = \int_z P(z)\, P(x|z)$$

$x \sim N(\mu, \sigma^2)$

$\mu = f(z)$

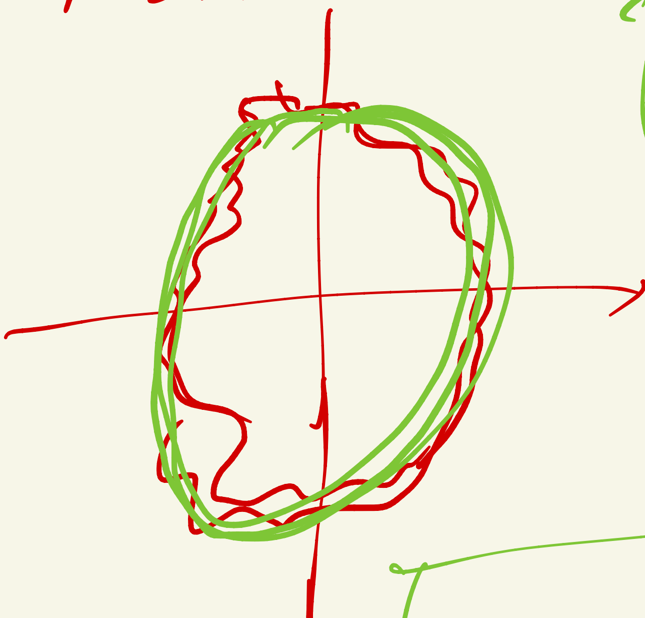
$$Q(z) = \boxed{P(z|x)} \propto \boxed{P(z)}\, \boxed{P(x|z)}$$

Gaussian Gaussian

$P(z|x)$ is not Gaussian

$$\boxed{E_{z \sim Q(z)} \log P(x, z; \theta)}$$

$p(z|x)$



$q(z|x) \sim \text{Gaussian}(\mu, \sigma^2)$

$\mu \quad \sigma^2$

distance $\mathcal{E}[p(z|x;\theta), q(z|x;\phi)]$

$$\underbrace{\log P_\theta(x)}_{\text{unrelated to } Q} \geq ELBO$$

$$\boxed{Q(z) = P(z|x)}$$

$$\Downarrow$$

$$\log P_\theta(x) = ELBO$$

$$ELBO = \log P_\theta(x) - KL[q_\phi(z|x) \| P(z|x)]$$

$$\arg\max_\phi ELBO = \arg\min_\phi KL(q_\phi(z|x) \| P(z|x))$$

we cannot compute $p(z|x)$

$q(z|x) \longrightarrow p(z|x)$

exponential

$\underset{q(z|x)}{\arg\max}$ ELBO

Variational EM

$p(z|x)$

$$\arg\max_{\theta} \underbrace{E_{z \sim q(z|x;\phi)}} \log \frac{p(x, z; \theta)}{q(z|x;\phi)}$$

$$\boxed{z \sim q(z|x;\phi)}$$

$$\arg\max_{\phi} E_{z \sim q(z|x;\phi)} \log \frac{p(x, z)}{q(z|x)}$$

function of ϕ

$z = g_z$ - - - - - $\boxed{z \sim q(z|x;\phi)}$

$z^{(1)}$

$$\log \frac{p(x, z^{(1)})}{q(z^{(1)}|x)}$$

$\varepsilon \sim N(0, 1)$

$z = \mu + \sigma \odot \varepsilon$

element-wise multipli

what is the distribution of z

$z \sim N(\mu, \sigma)$

$q(z|x)$

$z = \mu + \sigma \odot \varepsilon$

$z \sim q(z|x;\phi)$

$$\boxed{z = g(x, \phi, \varepsilon)}$$

z discrete

not

$q(z|x) \sim$ Gaussian

$\boxed{\text{Gumbel softmax}}$

$$ELBO = \log P(x) - KL[q(z|x) \| P(z|x)]$$

$$E_{z\sim q(z|x)}[\log P(x|z)]$$

$$KL(q(z|x) \| P(z))$$

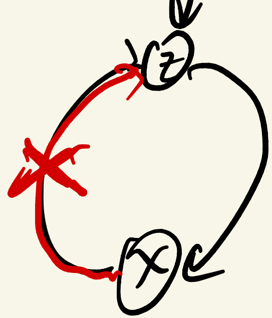
$$N(0,1)$$

$$\begin{tikzcd} z \arrow[d, bend left, "P(x|z)"] \\ x \arrow[u, bend left, "q(z|x)"] \end{tikzcd}$$

E-step $\quad q(z|x) \approx p(z|x) \quad \theta$

optimize $q(z|x;\phi)$ till convergence

$p(z)$



$p(z, x)$

$z \sim p(z)$

$x \sim p(x|z)$

VAE model $P(x|z)$, $P(z)$

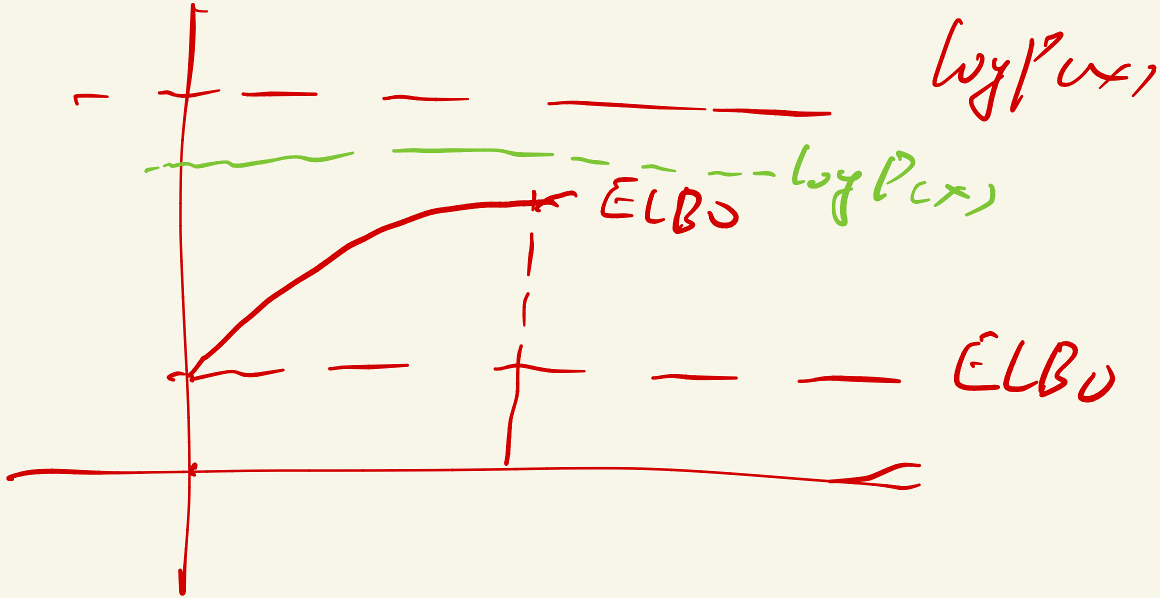
VAE inference

$$\mathcal{L} = ELBO$$

$\log P(x)$

E-step

$$q(z|x) \underset{\sim}{\longrightarrow} P(z|x)$$



$\boxed{P(x|z)}$ does not depend on θ practically

$$E_{z \sim q_\phi(z|x)}[\log P_\theta(x|z)]$$

$P(z) \sim N(0,1)$

autoencoder

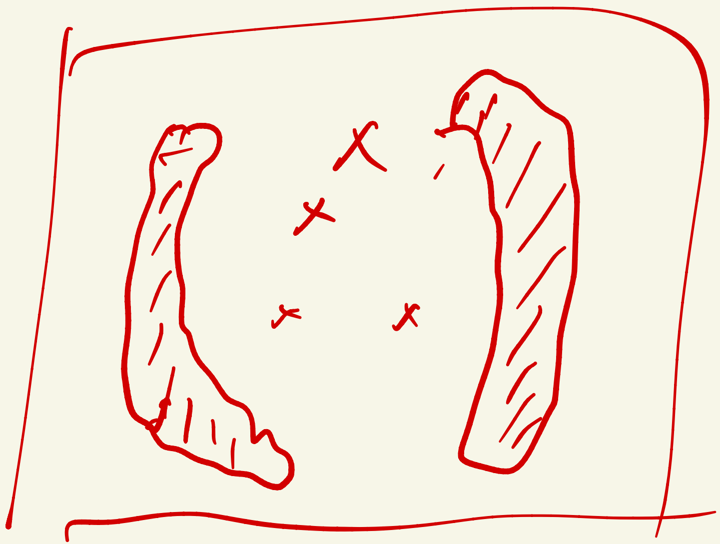
z

$\cancel{z}$ $\boxed{\text{how to sample } z}$

$\boxed{z \sim N(0,1)}$

$z \sim N(0,1)$

$\boxed{P(z)}$



$\boxed{P(x)}$

VAE:

$$x \sim N(\mu, \sigma^2)$$

$$\mu, \sigma^2 = f(z; \phi)$$

$N(0,1)$

$$P(x) = \int_z P(x|z) \, P(z)$$

GAN:

$$x = G(z)$$

$$x \sim N(\mu, \sigma^2)$$

$$\underbrace{\boxed{E_{z\sim q(z|x)} \underbrace{\log p(\cancel{x}|z)}}} \to -\infty$$

$$\underset{\sim}{z^{(1)}} \sim q(z|x)$$

$$\underset{\sim}{x}$$