Lecture 6 SVM

$$\max_{w,b} \underbrace{\frac{1}{\|w\|}}$$

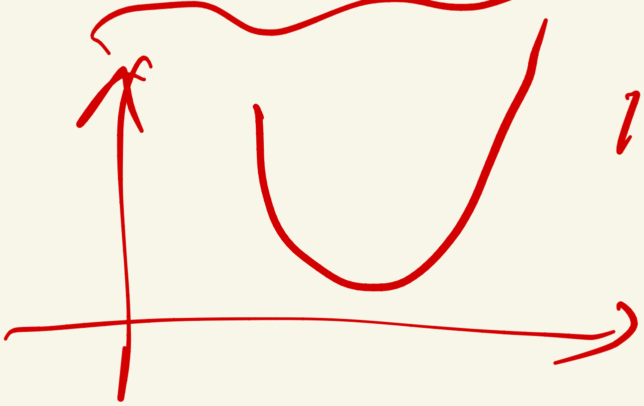
$$s.t. \quad y^{(i)}(w^T x^{(i)} + b) \geq 1$$

$$\underset{\sim}{2} \quad \underset{\sim}{0.5} \quad 2$$

equivalent

$$\underbrace{\underbrace{\min_{w,b} \frac{1}{2}\|w\|^2}}$$

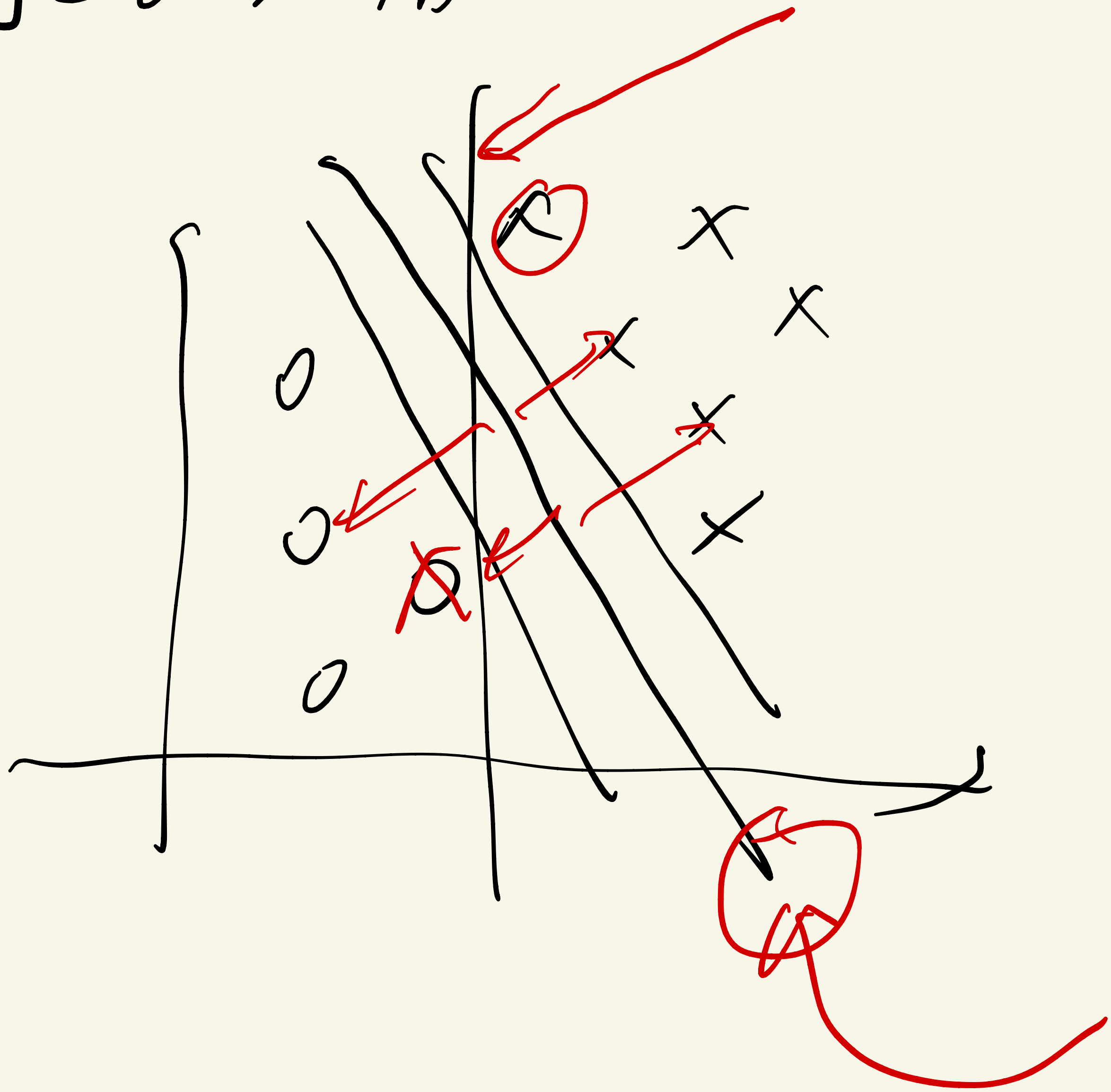
equivalent



$\|w\|^2$ $\quad \|w\|^2$

$\frac{1}{3}\|w\|^2$

$y = w^T x + b$



$$\gamma = \frac{\hat{\gamma}}{\|w\|} \quad \text{functional}$$

Geo

$$\underbrace{w^T x + b > 0}$$

$$y = w^T x + b$$

$$w^T x + b < 0$$

$$\hat{\gamma} = \underbrace{y(w^T x + b)}$$

$$\underbrace{w \rightarrow 2w}$$

$$\underbrace{b \rightarrow 2b}$$

$$\hat{\gamma} \rightarrow 2\hat{\gamma}$$

$$\begin{cases} 2w^T x + 2b > 0 \\ < 0 \end{cases}$$

$$\max_{w,b}\ \min_{i=1,\dots,n} \gamma^{(i)}$$

$$\Downarrow$$

$$\max_{\gamma,w,b}\ \gamma$$

$$\text{s.t. } y^{(i)}\left(\left(\frac{w}{\|w\|}\right)^T x^{(i)} + \frac{b}{\|w\|}\right) \geq \gamma$$

$$\frac{w}{\|w\|}\quad \gamma$$

$$\gamma = \frac{\hat{\gamma}}{\|w\|}$$

$$\boxed{\max_{w,\hat{\gamma},b} \frac{\hat{\gamma}}{\|w\|} \quad s.t.\ y^{(i)}(w^T x^{(i)}+b) \ge \hat{\gamma}}$$

$$\frac{\hat{\gamma}}{\|w\|} = \frac{\hat{\gamma}}{\sqrt{w^T w}}$$

$$\frac{\hat{\gamma}}{\|w\|} \quad \frac{\hat{\gamma}_0}{\|w_0\|} = \frac{2\hat{\gamma}_0}{\|2w_0\|}$$

$$\begin{cases} w_0 & b_0, & \hat{\gamma}_0 \\ \downarrow & \downarrow & \\ 2w_0 & 2b_0 & 2\hat{\gamma}_0 \end{cases}$$

$$y^{(i)}(w_0^T x^{(i)} + b_0) \ge \hat{\gamma}_0$$

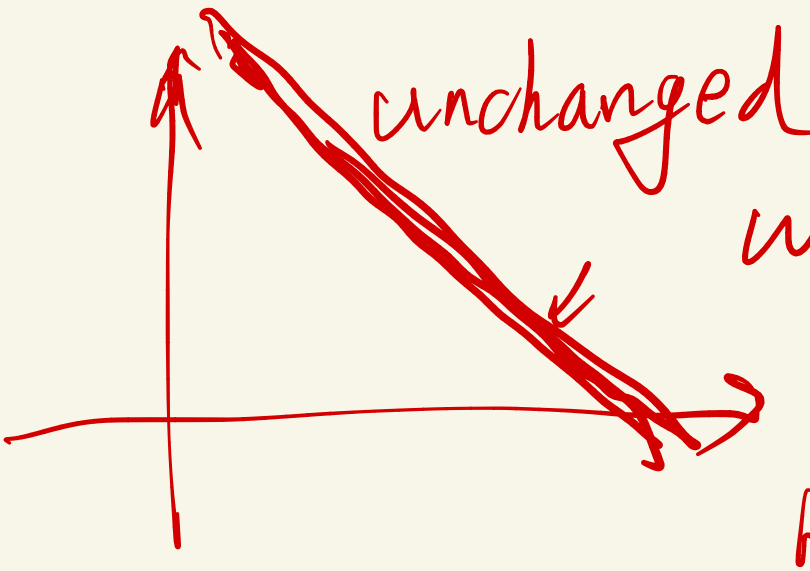
$$y^{(i)}(2w_0^T x^{(i)} + 2b_0) \ge 2\hat{\gamma}_0$$

under regularized

$$\boxed{\|w\| = 1} \longrightarrow \|w\| = 1$$

$= 0.5$

$= 2$



$w \to 2w \to 4w$

$\to 8w$

$b \to 2b \to 4b$

$$\frac{\hat{\gamma}}{\|w\|} \qquad \hat{\gamma} = 1$$

$$\downarrow$$

$$\max \frac{1}{\|w\|}$$

$$\Big\Downarrow \text{ equivalent}$$

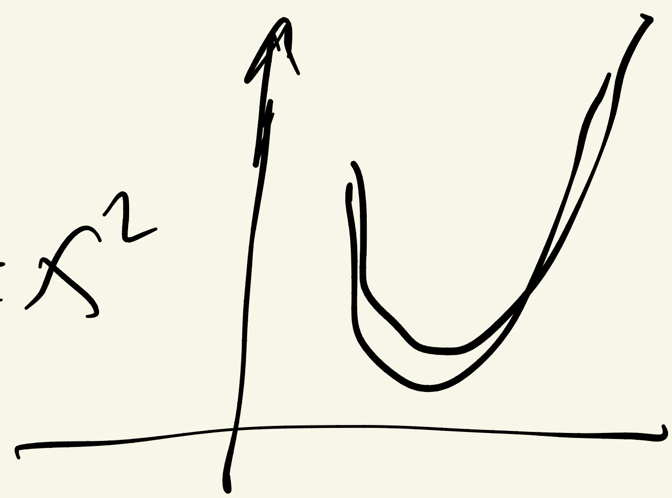
$$\min \|w\| \longrightarrow \min \|w\|^2$$

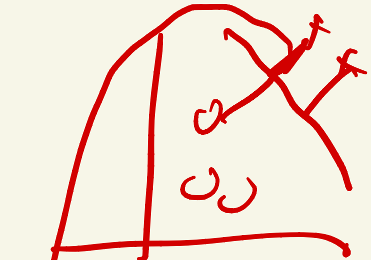
$$\arg\max_w \frac{1}{\|w\|} = \arg\min_w \|w\| = \arg\min_w \|w\|^2$$

$$= \arg\min_w \frac{1}{2}\|w\|^2$$

$$\|w\|^2$$

$y = x^2$





$$\frac{\max \frac{1}{2}\|w\|^2}{y^{(i)}(w^T x^{(i)} + b) \geq 1}$$

$$\gamma^{(i)} = \frac{y^{(i)}(w^T x^{(i)} + b)}{\|w\|}$$

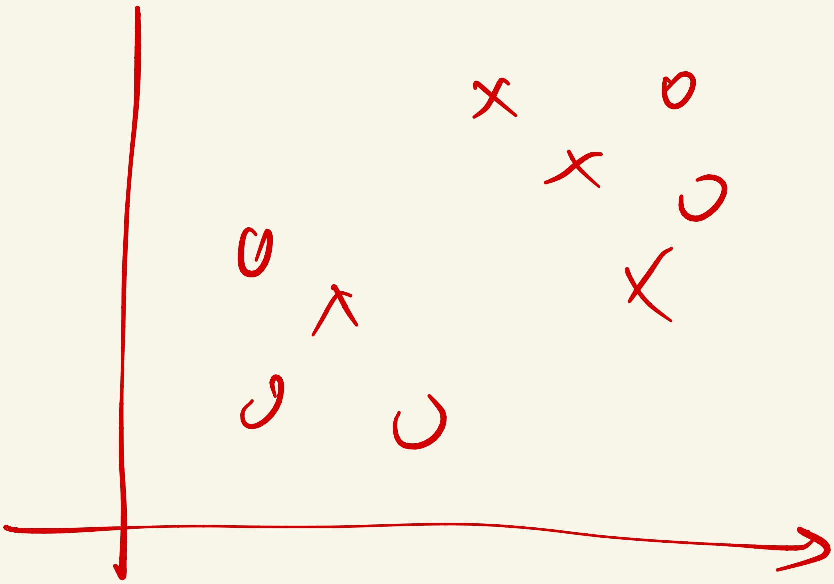
3

2

$$\boxed{y^{(i)}(w^T x^{(i)} + b) \geq 1}$$

$$\gamma = \frac{\min y^{(i)}(w^T x^{(i)} + b) = 1}{\|w\|}$$

$$\max \frac{1}{\|w\|} \xleftrightarrow{\text{equivu}} \min \frac{1}{2}\|w\|^2$$



$$y^{(i)}(w^T x^{(i)} + b) \geq 1$$

$$< 0$$

linear svm

$$y^{(i)}(w^T x^{(i)} + b) \geq 1$$

$$\phi(x) = \begin{bmatrix} x_1 \\ x_1^2 \\ x_2^2 x_1 \\ \vdots \\ x \end{bmatrix}$$

high dim

$$y^{(i)}(w^T \phi(x)^{(i)} + b) \geq 1$$

$\phi(x)$

$$(y^{(i)} - \theta^T x^{(i)})\, x^{(i)}$$

$$\theta^T = \sum_i \beta_i x^{(i)}$$

$$K(x, z) = (x^T z)^2$$

$$\frac{\partial L}{\partial \beta_i} = 0$$

$$\underbrace{\frac{\partial L}{\partial \beta_i}} = \underbrace{h_i(w)} = 0$$

$$\beta = \pm \frac{1}{4}$$

before: w $\quad g_i(w) \le 0$

after: α, β $\quad w \to h_i(w) = 0$

> 0

0

$$\underset{\text{primal}}{\theta_p(w)} = \max_{\alpha, \beta, \alpha \geq 0} f(w) + \sum_{i=1}^{k} \alpha_i \boxed{g_i(w)} + \sum_{i=1}^{l} \beta_i h_i(w)$$

$\neq 0$

$$\min_{w} f(x)$$

$$g_i(w) \le 0 \qquad \Rightarrow \qquad \min_{w} \max_{\alpha, \beta, \alpha \ge 0} L(w, \alpha, \beta)$$

$$h_i(w) = 0$$

(no constraint ?)

$$\boxed{\theta_p(w)} \qquad\qquad \min_{w} f(x)$$

$$p^* = \min_{w} \theta_p(w) = \min_{w} f(w)$$

$$\text{s.t. } g_i(w) \le 0$$

$$h_i(w) = 0$$

$$\min_{w,b} \frac{1}{2}\|w\|^2$$

$$\text{s.t. } y^{(i)}(w^T x^{(i)} + b) \geq 1 \quad i = 1, \ldots, n$$

$$\theta_D(\alpha, \beta) = \min_{w} L(w, \alpha, \beta) \rightarrow \text{dual}$$

$$\theta_p(w) = \max_{\alpha \geq 0, \beta} L(w, \alpha, \beta) \rightarrow \text{original}$$

$$\downarrow_{w}$$

$$\min \theta_p(w) \longrightarrow \text{original}$$