Lecture 7 SVM

$\gamma^{(i)}$ geometric margin

$\|w\|$

$\hat{\gamma}$ functional margin

$$\gamma = \frac{\hat{\gamma}}{\|w\|}$$

$$\hat{\gamma} = y(w^T x + b)$$

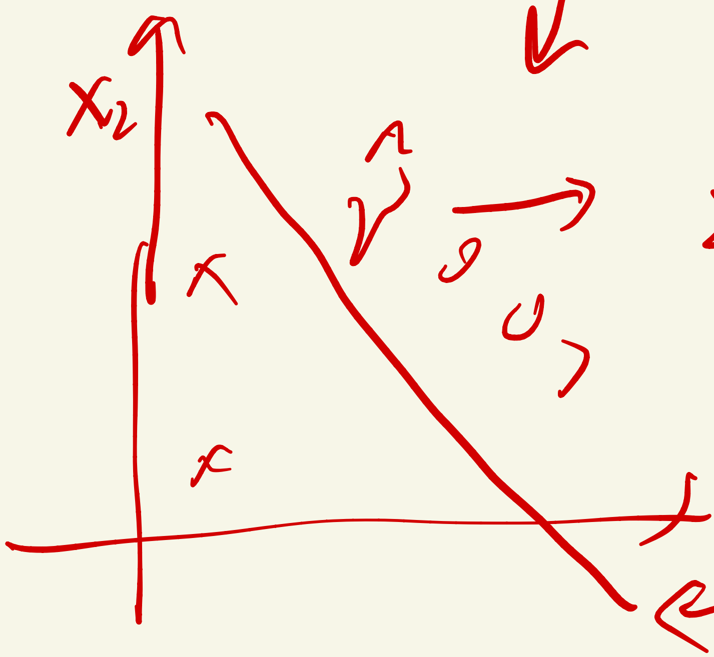
$$w \longrightarrow 2w$$

$$b \longrightarrow 2b$$

$$\hat{\gamma} \longrightarrow 2\hat{\gamma}$$

$$\cancel{2}w^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \cancel{2}b = 0$$

$$w^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b = 0$$

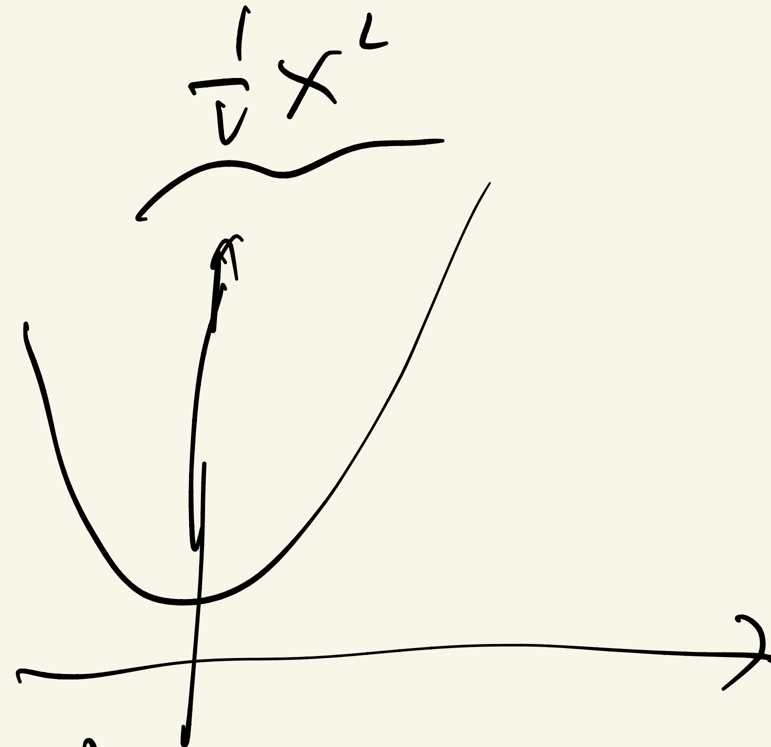


$$\|w\| = 1,\ 2$$

$$\downarrow$$

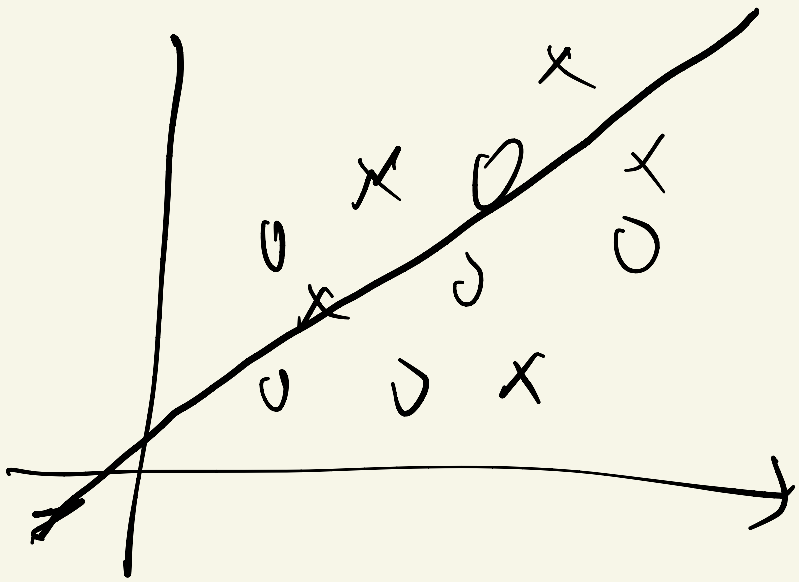
$$\hat{\gamma} = 1$$

$$\hat{\gamma} = 1$$



$$\frac{\hat{\gamma}}{\|w\|} \longrightarrow \frac{1}{\|w\|}$$

$$\arg\max_{w} \frac{1}{\|w\|} = \arg\min_{w} \frac{1}{2}\|w\|^2$$



$$\boxed{y^{(i)}(w^T x^{(i)} + b)} \geq 1$$

< 0

$$L(w, \beta) = f(w) + \sum_{i=1}^{l} \beta_i h_i(w)$$

$$\frac{\partial L}{\partial \beta_i} = h_i(w) = 0$$

l

$\neq 0$

$$f(w) + \sum_{i=1} \alpha_i g_i(w) \leq 0$$

0

$g_i(w) > 0$

$\alpha_i \geq 0$

$$\theta_p(w) = \begin{cases} f(w) & \text{if } w \text{ satisfies constraints} \\ \infty & \text{otherwise} \end{cases}$$

$$\boxed{\min_w \theta_p(w)} \equiv \min_w f(w)$$

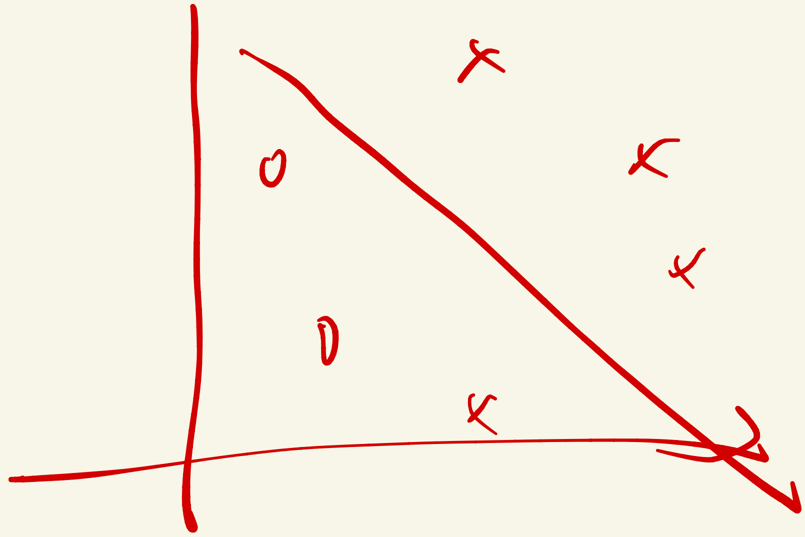
under the primal constr

$$p^* = \min_w f(w)$$

under th— --

$w^T x$

$w^T \phi(x)$



$$\phi(x) = \begin{bmatrix} x_1 \\ x_1^2 x_2 \\ x_2^2 \\ x_2^3 \\ \vdots \end{bmatrix}$$ ← no need

$$k(x_1, x_2) = \langle \phi(x_1), \phi(x_2) \rangle$$

$$k(x, z) = (x^T z)^k \quad d$$

$\phi(x) \to d^k$

$\theta_p(\omega)$ func of ω

$\theta_D(\alpha, \beta)$ func of $\alpha . \beta$

$$d^* \le p^*$$

$$\max_x \min_y f(x,y) \le \min_y \max_x$$

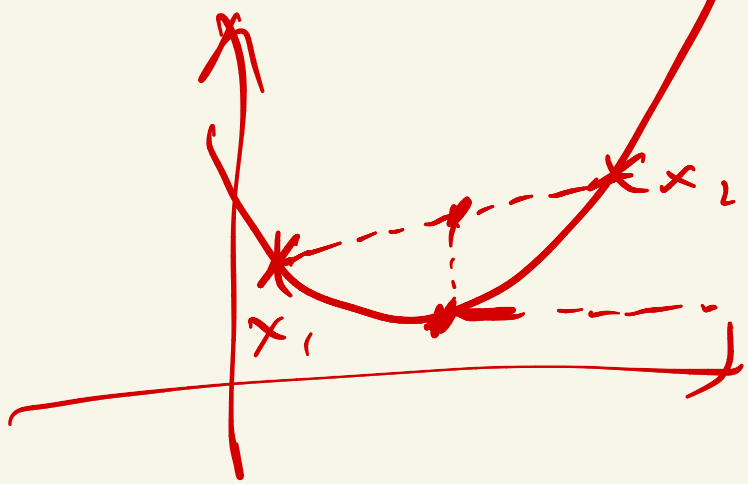
convex function def:

$f(x, y)$

$f(x)$

$0 \le \alpha \le 1$

$$f(\alpha x_1 + (1-\alpha) x_2) \le \alpha f(x_1) + (1-\alpha) f(x_2)$$



$g_i(w) < 0$

SVM:

$$f(w) = \frac{1}{2}\|w\|^2$$

$$g(w) = 1 - y(w^T x + b)$$

$$y(w^T x + b) \geq 1$$

$$g(w) \leq 0$$

$p^* = d^*$

slater condition $\Rightarrow$ strong duality

strong duality $\Longleftrightarrow$ KKT

$\alpha_i^* g_i(w_i^*)$

solution to $\theta_D(\alpha, \beta)$

w^* α_i^* β_i^*

solution to $\theta_p(w)$

$$\boxed{\alpha_i^* g_i(w^*) = 0}$$

$g_i(w^*) \leq 0$

$g_i(w^*) < 0 \longrightarrow \alpha_i = 0$

g

$$y^{(i)}(w^T x^{(i)} + b) \geq 1$$

$$g_i(w^*) = 1 - y^{(i)}(w^T x^{(i)} + b) \leq 0$$

$$= 0$$

$$< 0$$

$$\alpha_i^* g_i(w^*) = 0$$

$$\boxed{\sum_{i=1}^{n} \alpha_i y^{(i)} = 0}$$

$L(w, b, \alpha)$

$\theta_D(\alpha)$

$$\max_{w} \frac{1}{2}\|w\|^2 \longrightarrow \max_{\alpha}$$

$$\langle x^{(i)}, x^{(j)} \rangle = x^{(i)T} x^{(j)}$$

$\downarrow$

$K[x^{(i)}, x^{(j)}]$

$$\min y^{(i)}(w^{*T}x^{(i)}+b^*)=1$$

~~max~~

$y^{(i)}=1$

$\min w^{*T}x$

$y^{(i)}=-1$

$\max w^{*T}x$

$$\phi(x)$$

3 vectors

$$\sum_{i=1}^{n} \alpha_i y^{(i)} K(x^{(i)}, x) + b$$

3 items

$$\frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n} \xi_i$$

ξ_i

$\underbrace{\geq 1 - \xi_i}$ $\boxed{\xi_i \geq 0}$