香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

COMP 5212
Machine Learning
Lecture 7

# Generative Models, Naive Bayes

Junxian He
Mar 5, 2026

# Discriminative vs. Generative Learning

# Discriminative vs. Generative Learning


X

$p(y \mid x)$

Discriminative

Cat Y

# Discriminative vs. Generative Learning

X

Cat Y

$p(y)$

Generative

$p(x \mid y)$

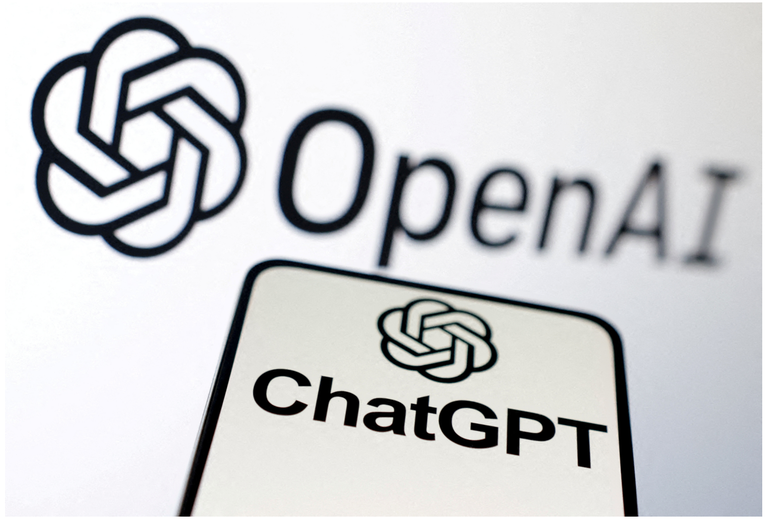
$p(y \mid x)$

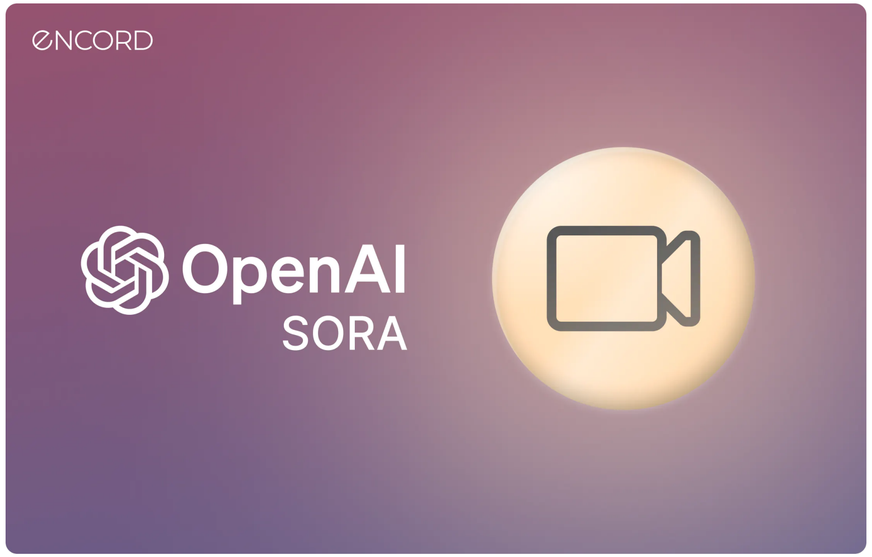
Discriminative

Cat Y

X

# Generative Model Examples




$x$

$p(x)$

# Video Generation Examples

Prompt: A stylish woman walks down a Tokyo street filled with warm glowing neon and animated city signage. She wears a black leather jacket, a long red dress, and black boots, and carries a black purse. She wears sunglasses and red lipstick. She walks confidently and casually. The street is damp and reflective, creating a mirror effect of the colorful lights. Many pedestrians walk about.

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Discriminative

Cat Y

Cat Y $p(y)$

x sample

Generative

$p(x \mid y)$

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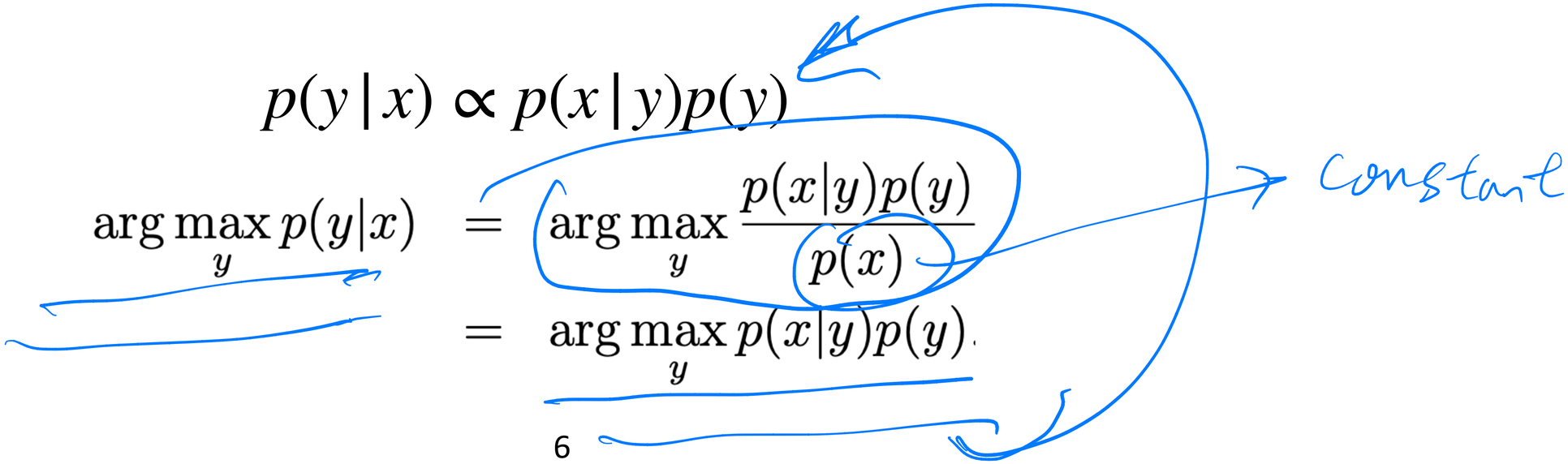
$$\arg\max_y p(y|x) = \arg\max_y \frac{p(x|y)p(y)}{p(x)} = \arg\max_y p(x|y)p(y)$$



# Generative Models Compared to Discriminative Models

Pros:

Cons:

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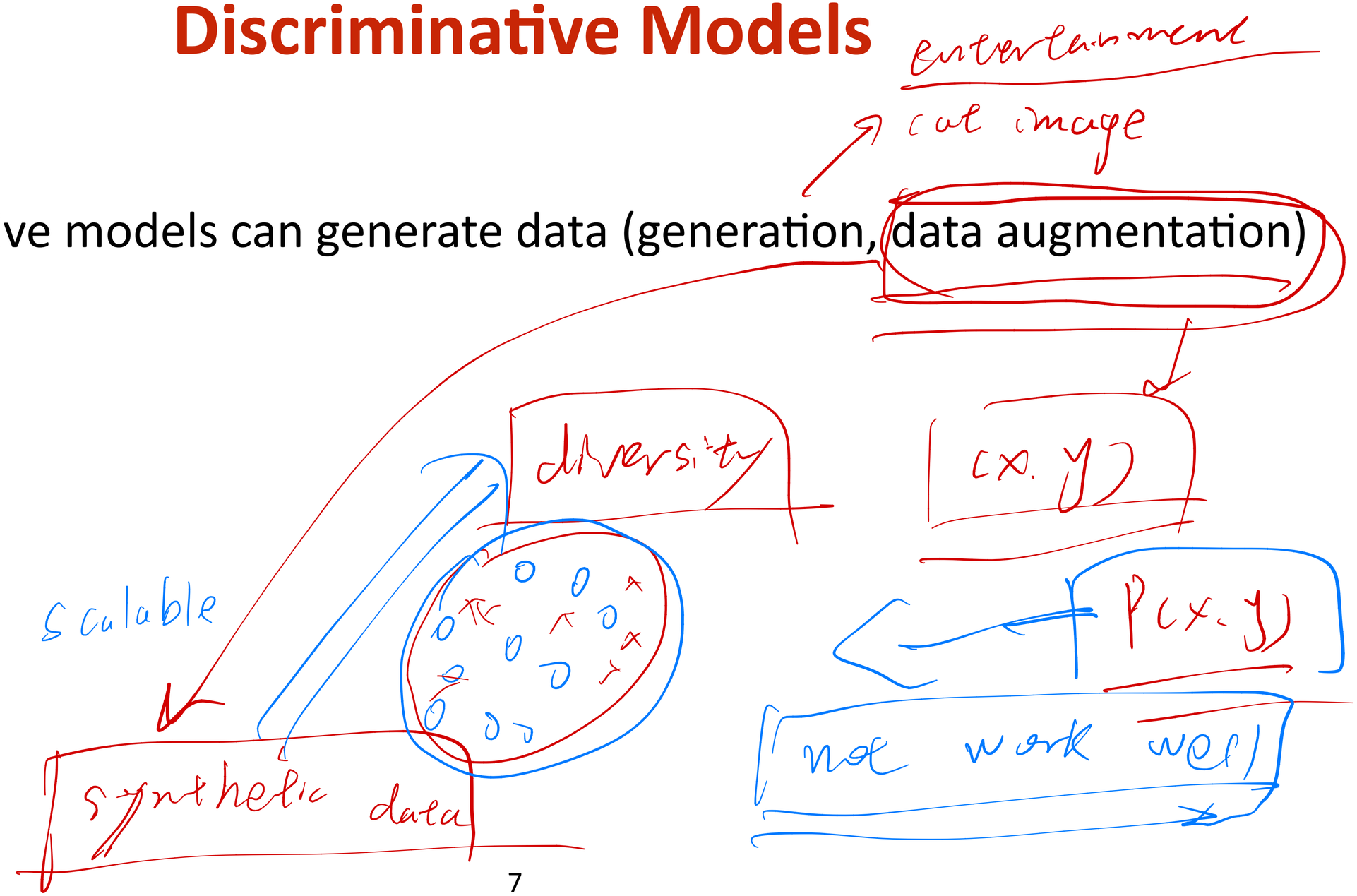
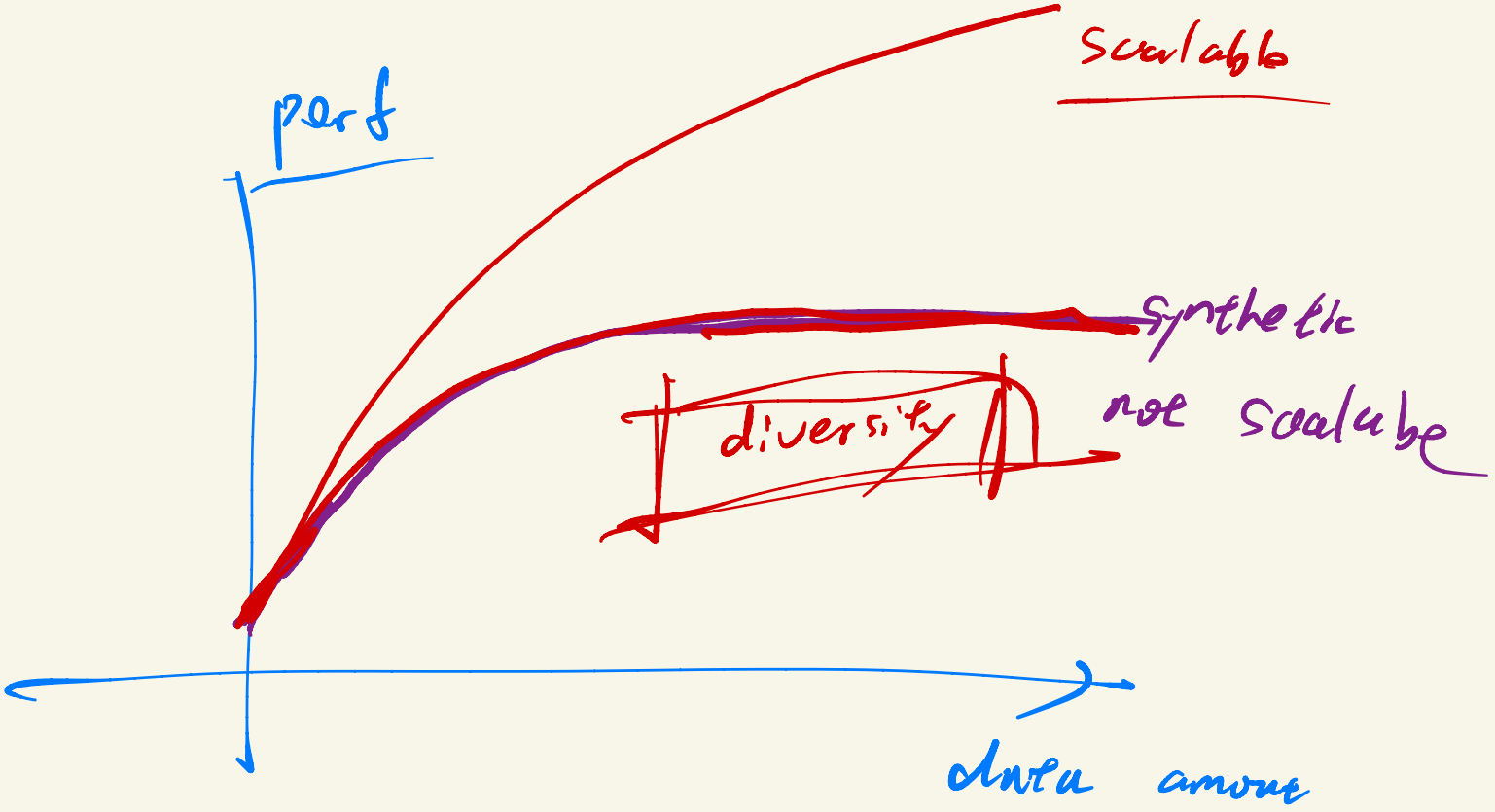
Pros:

- Generative models can generate data (generation, data augmentation)

Cons:




Scalable
perf
Synthetic
diversity
not scalable
data amount

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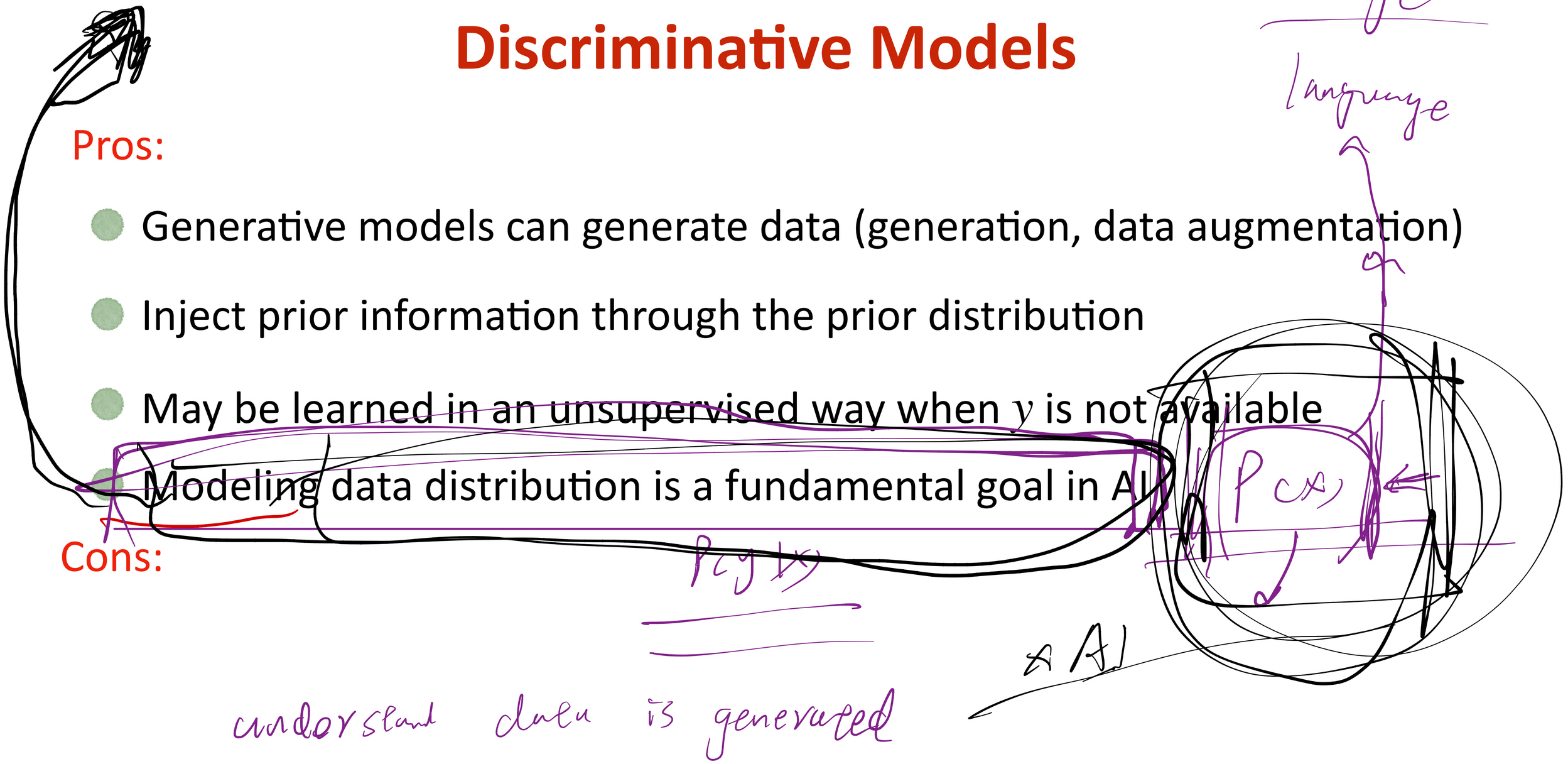

Pros:

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- Inject prior information through the prior distribution
- May be learned in an unsupervised way when $y$ is not available
- Modeling data distribution is a fundamental goal in AI

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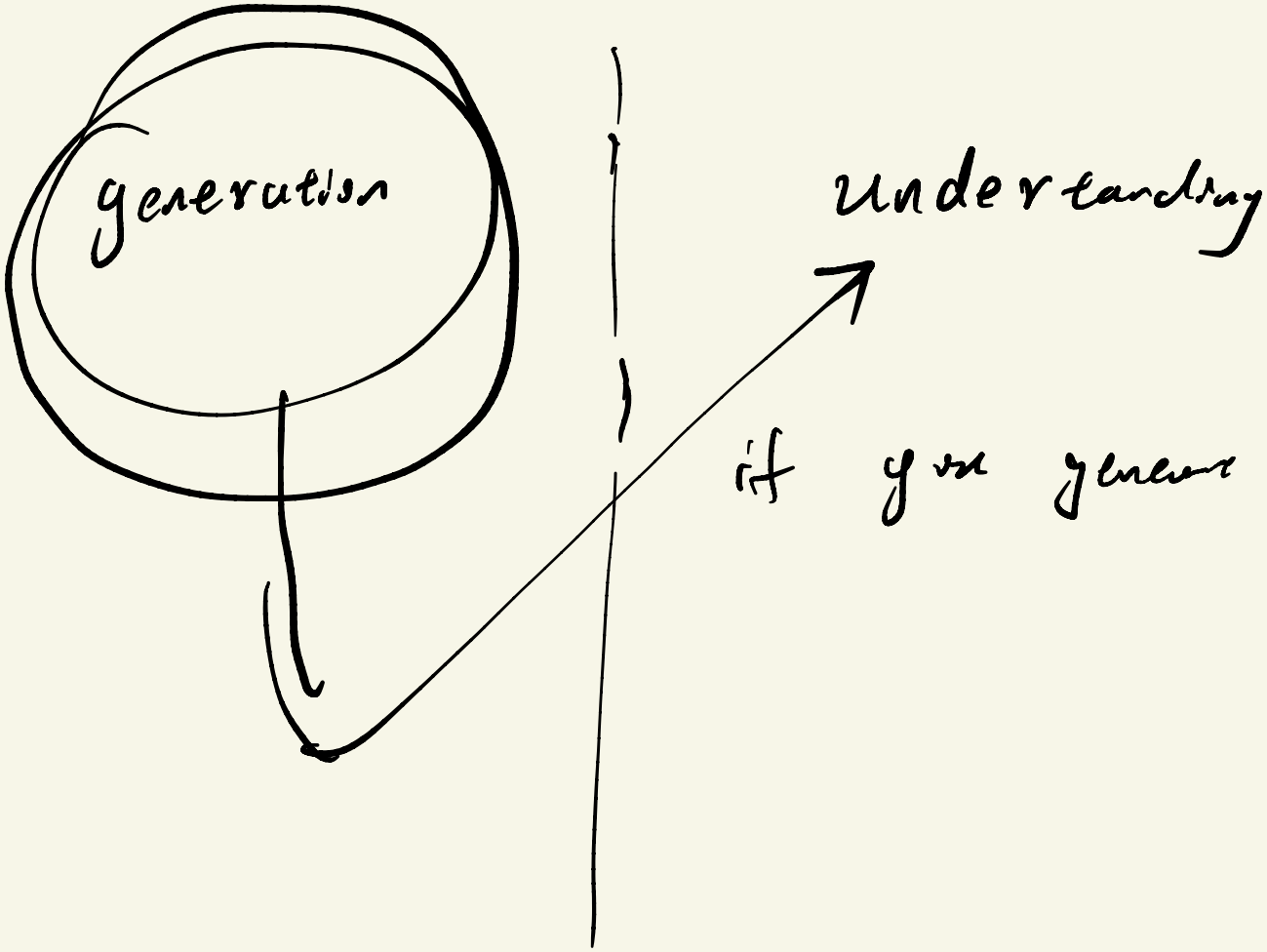
generation
Understanding
if you generate

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Cons:

$(x, y)$

- Often underperforms discriminative models on discriminative tasks because of stronger assumptions on the data

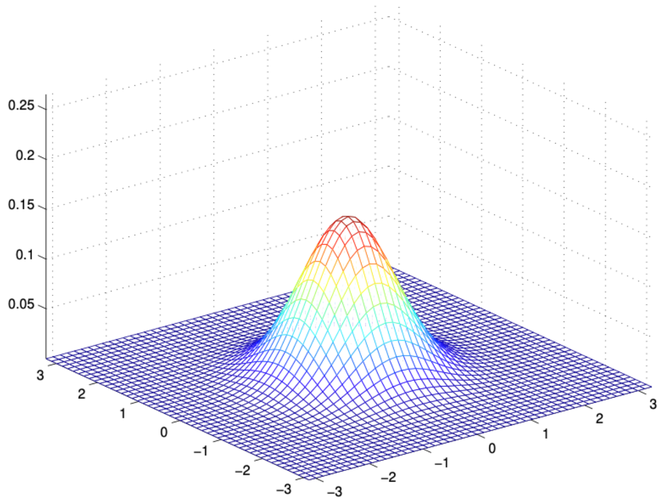
# Gaussian Discriminant Analysis Model (GDA)

Multivariate Gaussian distribution

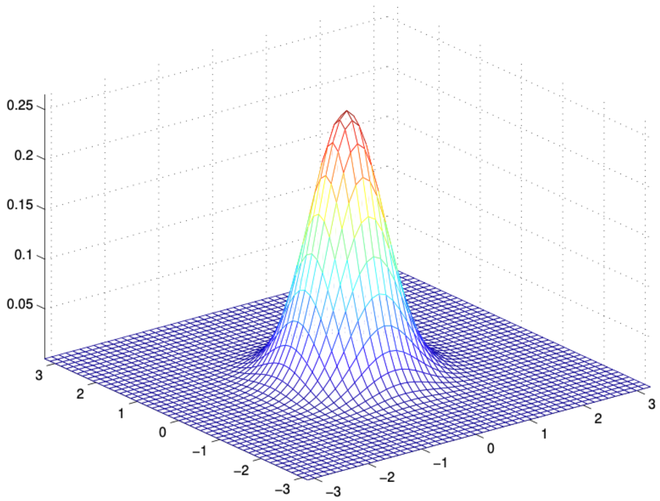
$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

- $\Sigma \in R^{dxd}$ is the covariance matrix, it is also symmetric positive semi-definite
- $|\Sigma|$ denotes the determinant of $\Sigma$

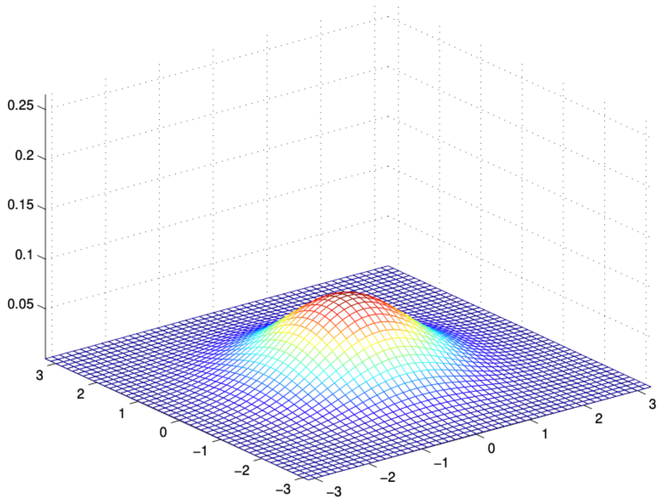
# Examples of Multivariate Gaussian




$\Sigma = I$

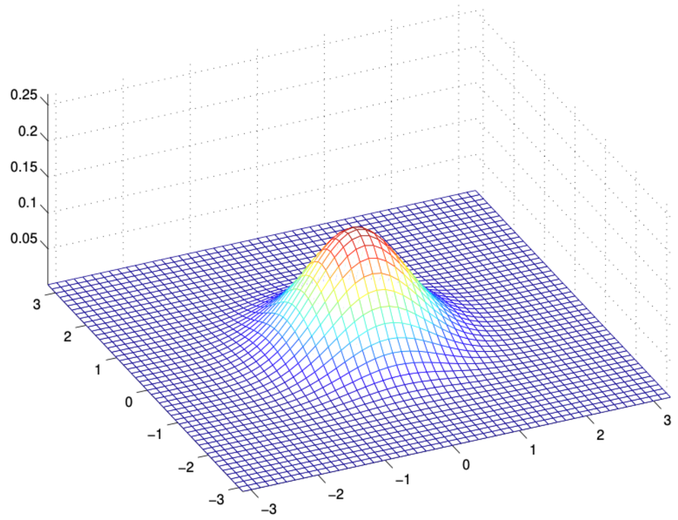



$\Sigma = 0.6I$

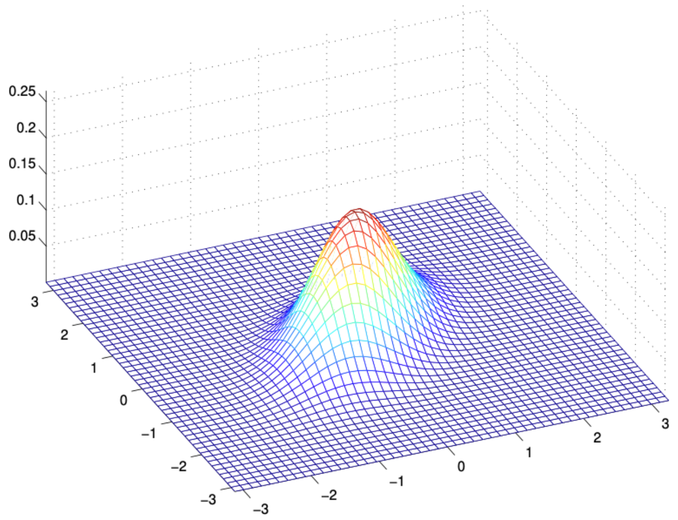



$\Sigma = 2I$

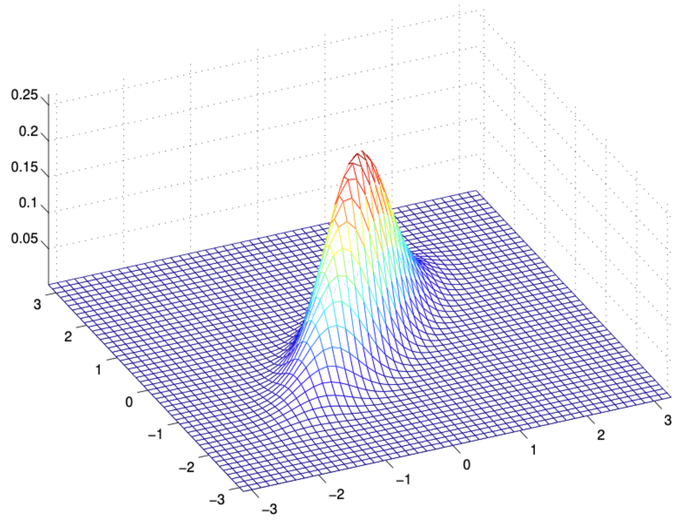
# Examples of Multivariate Gaussian




$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$




$$\Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$




$$\Sigma = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

# Gaussian Discriminant Analysis Model

Binary classification: $y \in \{0,1\}, x \in R^d$

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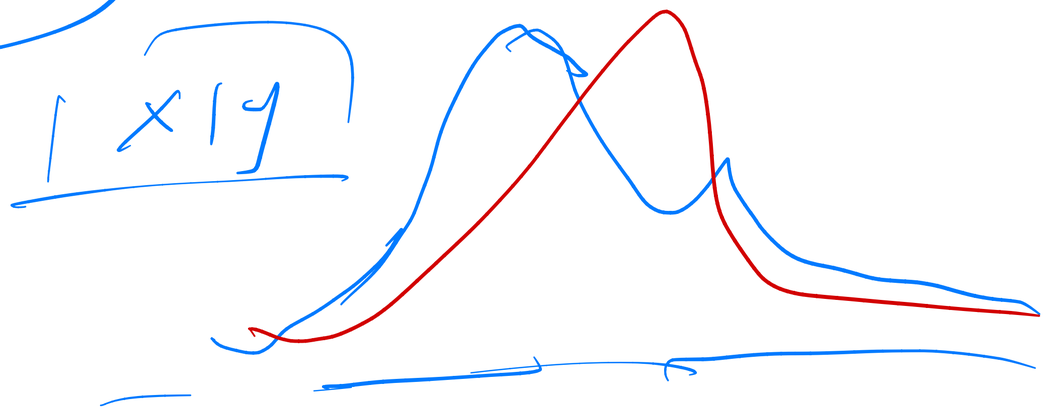
Assumption

$$
\begin{aligned}
y &\sim \text{Bernoulli}(\phi) \\
x|y=0 &\sim \mathcal{N}(\mu_0, \Sigma) \\
x|y=1 &\sim \mathcal{N}(\mu_1, \Sigma)
\end{aligned}
$$



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$$

$$
\begin{aligned}
p(y) &= \phi^y (1-\phi)^{1-y} \\
p(x|y=0) &= \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu_0)^T\Sigma^{-1}(x-\mu_0)\right) \\
p(x|y=1) &= \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right)
\end{aligned}
$$

# Maximum Likelihood Estimation

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$$\ell(\phi, \mu_0, \mu_1, \Sigma) = \log \prod_{i=1}^{n} p(x^{(i)}, y^{(i)}; \phi, \mu_0, \mu_1, \Sigma)$$

$$= \log \prod_{i=1}^{n} p(x^{(i)} | y^{(i)}; \mu_0, \mu_1, \Sigma) p(y^{(i)}; \phi).$$

argmax $\ell$

$\phi, \mu_0$

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$$\phi = \frac{1}{n} \sum_{i=1}^{n} 1\{y^{(i)} = 1\}$$

$$\mu_0 = \frac{\sum_{i=1}^{n} 1\{y^{(i)} = 0\} x^{(i)}}{\sum_{i=1}^{n} 1\{y^{(i)} = 0\}}$$

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$$\Sigma = \frac{1}{n} \sum_{i=1}^{n} (x^{(i)} - \mu_{y^{(i)}})(x^{(i)} - \mu_{y^{(i)}})^T$$

# Maximum Likelihood Estimation

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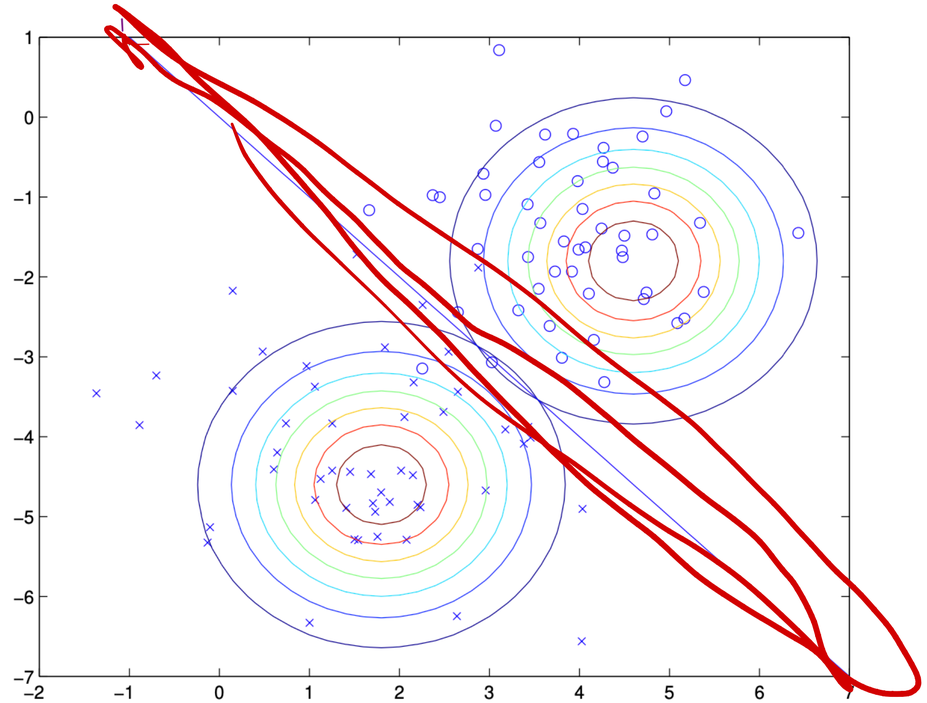

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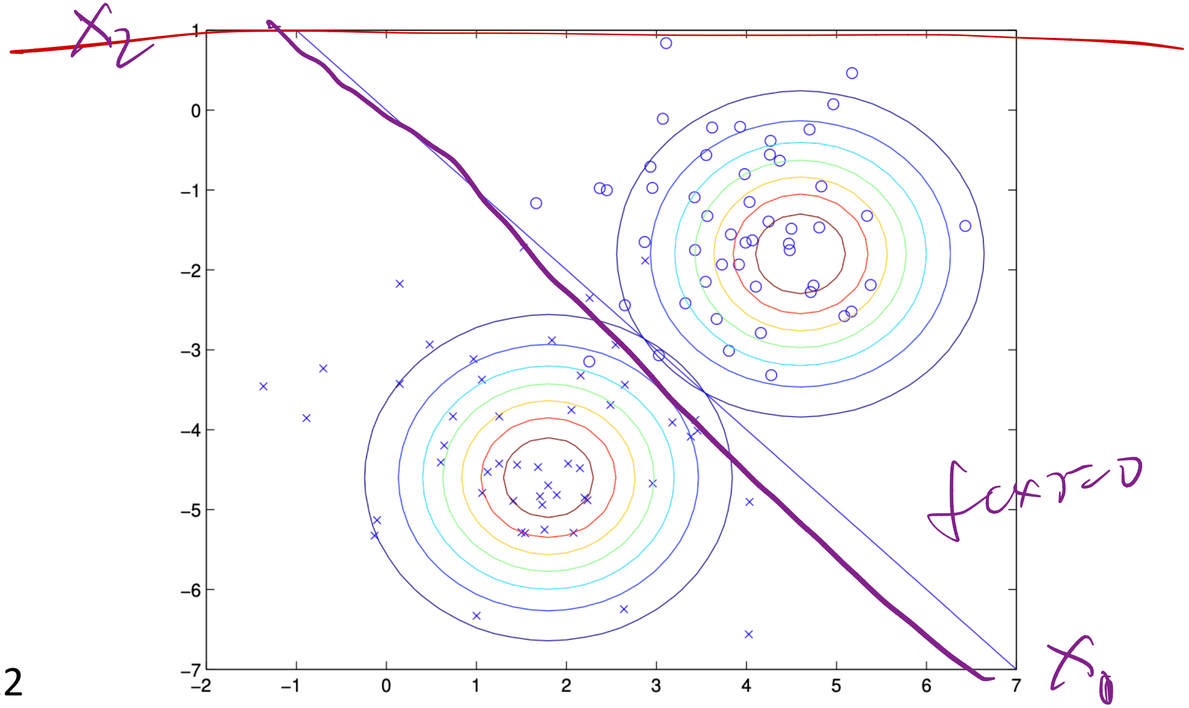
Why is the decision boundary linear?



decision boundary

$$P(y|x) \propto P(y)\,P(x|y)$$

$$P(y=1|x) = P(y=0|x)$$

$$f(x) = 0$$ decision boundary

$$\log P(y=1|x) = \log P(y=0|x) \Rightarrow$$

# Connection Between GDA and Logistic Regression

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Through Bayes rule, we can show that

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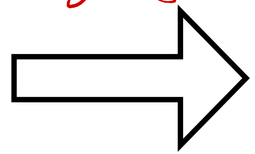
assumption

$p(x \mid y)$ is Gaussian



sufficient

$p(y \mid x)$ follows logistic regression

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more flexible

$p(x \mid y)$ is Gaussian $\Rightarrow$ $p(y \mid x)$ follows logistic regression

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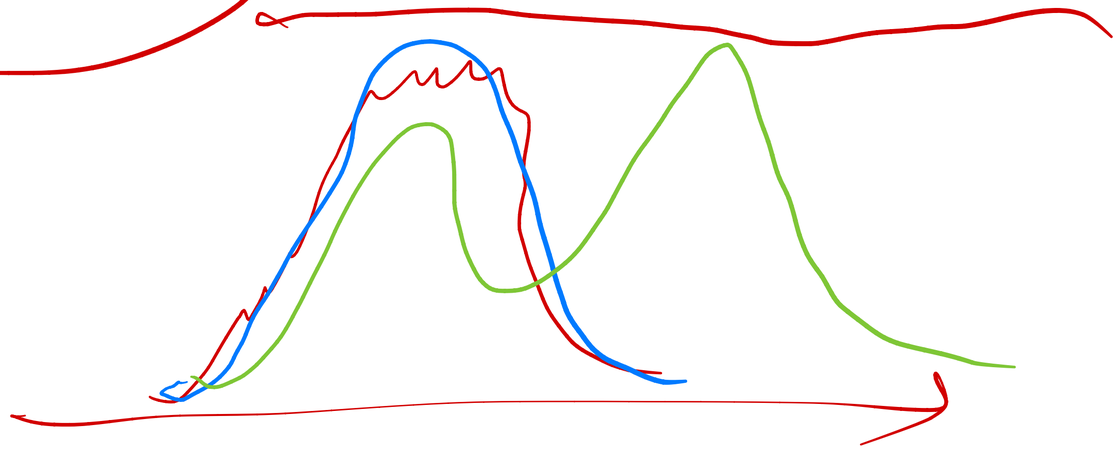
- When $x|y$ does not follow Gaussian in practice, GDA may or may not do well



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These are intuitions generally applicable to machine learning

# Philosophy Behind Modeling Assumptions / Priors

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2. The famous and bitter lesson from IBM machine translation model: “Every time I fire a linguist, the model performance goes up” — Frederick Jelinek

# The Bitter Lesson

http://www.incompleteideas.net/IncIdeas/BitterLesson.html

“The biggest lesson that can be read from 70 years of AI research is that general methods that leverage computation are ultimately the most effective, and by a large margin” — Rich Sutton

# Naive Bayes

Binary classification: $y \in \{0,1\}$, $x$ is discrete

$P(x|y)$

$P(y|x)$

if an email contains the j-th word of the dictionary, then we will set $x_j = 1$; otherwise, we let $x_j = 0$

x email

# Naive Bayes

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Consider an email spam detection task, to predict whether the email is spam or not

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$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \text{a} \\ \text{aardvark} \\ \text{aardwolf} \\ \vdots \\ \text{buy} \\ \vdots \\ \text{zygmurgy} \end{matrix}$$

size of dict

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Dimension is the size of the dictionary

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vocabulary

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Suppose the dictionary has 50000 words, how many possible x?

$P(x|y)$

$P(x=x_0|y) = prob(x_0)$

parameter

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Naive Bayes assumption: $x_i$'s are conditionally independent given $y$

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Suppose the dictionary has 50000 words, how many possible x?

$P(x|y) = P(x_1|y) P(x_2|y) \cdots P(x_n|y)$

Naive Bayes assumption: $x_i$'s are conditionally independent given $y$

For any i and j, $p(x_i | y) = p(x_i | y, x_j)$

computer

software

# Email Spam Classification

# Email Spam Classification

chain rule

$$
\begin{aligned}
& p(x_1, \ldots, x_{50000}|y) \\
&= p(x_1|y)p(x_2|y,x_1)p(x_3|y,x_1,x_2)\cdots p(x_{50000}|y,x_1,\ldots,x_{49999}) \\
&= p(x_1|y)p(x_2|y)p(x_3|y)\cdots p(x_{50000}|y) \\
&= \prod_{j=1}^{d} p(x_j|y)
\end{aligned}
$$

# Email Spam Classification

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Autoregressive

# Email Spam Classification

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$$

Autoregressive

Parameters

$$\phi_{j|y=1} = p(x_j = 1 \,|\, y = 1), \quad \phi_{j|y=1} = p(x_j = 1 \,|\, y = 0), \quad \phi_y = p(y = 1)$$

$P(x_j=0|y=1) = 1 - P(x_j=1|y=1)$

$P(y=0)$

# Email Spam Classification

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\begin{aligned}
& p(x_1, \ldots, x_{50000} | y) \\
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50000 x 2 + 1 parameters (dict size is 50000)

# Maximum Likelihood Estimation

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$$\mathcal{L}(\phi_y, \phi_{j|y=0}, \phi_{j|y=1}) = \prod_{i=1}^{n} p(x^{(i)}, y^{(i)})$$

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Count the occurrence of $x_j$ in spam/ non-spam emails and normalize

# Prediction

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$$p(y=1|x) = \frac{p(x|y=1)p(y=1)}{p(x)}$$

$$= \frac{\left(\prod_{j=1}^{d} p(x_j|y=1)\right) p(y=1)}{\left(\prod_{j=1}^{d} p(x_j|y=1)\right) p(y=1) + \left(\prod_{j=1}^{d} p(x_j|y=0)\right) p(y=0)}$$

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Naive Classifier

$P(y=1|x) \propto P(x|y=1)\ P(y=1)$

$P(x|y=1)\ P(y=0)$

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Suppose the index in the dictionary for “learning” is q

$$p(x_q = 1 \mid y = 1) = 0$$

$$p(x_q = 1 \mid y = 0) = 0$$

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$$= \frac{\left(\prod_{j=1}^{d} p(x_j|y = 1)\right) p(y = 1)}{\left(\prod_{j=1}^{d} p(x_j|y = 1)\right) p(y = 1) + \left(\prod_{j=1}^{d} p(x_j|y = 0)\right) p(y = 0)}$$

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Take the problem of estimating the mean of a multinomial random variable $z$ taking values in {1, …, k}. Given the independent observations $\{z^{(1)}, \cdots, z^{(n)}\}$

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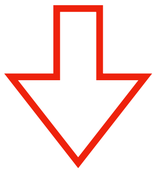

$$\phi_j = \frac{1 + \sum_{i=1}^{n} 1\{z^{(i)} = j\}}{k + n}$$

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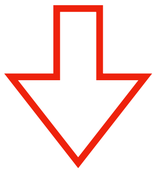

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Why adding k to the denominator?

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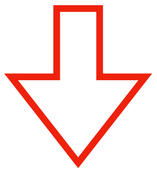

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Why adding k to the denominator?

In the email spam classification case:

$$\phi_{j|y=1} = \frac{1 + \sum_{i=1}^{n} 1\{x_j^{(i)} = 1 \wedge y^{(i)} = 1\}}{2 + \sum_{i=1}^{n} 1\{y^{(i)} = 1\}}$$

$$\phi_{j|y=0} = \frac{1 + \sum_{i=1}^{n} 1\{x_j^{(i)} = 1 \wedge y^{(i)} = 0\}}{2 + \sum_{i=1}^{n} 1\{y^{(i)} = 0\}}$$

# Thank You!
# Q & A